Fragility of Resale Markets for Securitized Assets and Policy of Asset Purchases

Martin Kuncl[*]

Bank of Canada

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Abstract

Markets for securitized assets were characterized by high liquidity prior to the recent financial crisis and by a sudden market dry up at the onset of the crisis. A general equilibrium model with heterogeneous investment opportunities and information frictions predicts that, in boom periods or mild recessions, the degree of adverse selection in resale markets for securitized assets is limited thanks to the reputation-based guarantees by asset originators. This supports investment and output. However, in a deep recession, characterized by high dispersion of asset qualities, there is a sudden surge in adverse selection due to an economy-wide default on reputation-based guarantees, which persistently depresses the output in the economy. Government policy of asset purchases limits the negative effects of adverse selection on the real economy, but may create a negative moral hazard problem.

1 Introduction

In the decades preceding the late 2000s financial crisis, securitization grew significantly in importance as a means of financial intermediation (Adrian and Shin, 2009). Prior to the crisis, the markets for securitized assets were very liquid, risk premia were low and traded volumes were growing. This was despite the fact that a large quantity of low quality loans were issued and securitized (infamous examples were some of the subprime mortgages), and despite the complex and opaque nature of some of the securitized assets. But then during the summer of 2007, at the onset of the financial crisis, we could observe a sudden and severe market dry-up. Brunnermeier (2009) documents how risk premia for the mortgage backed securities (assets backed by pools of mortgages / MBS) rapidly increased and funding for securitization in the form of asset backed commercial papers (ABCP)[1] disappeared. This is illustrated in Figure 1.

Due to the negative role of securitization at the onset of the financial crisis (see e.g. Bernanke, 2010), a lot of the recent research studied the design of securitization, where information asymmetries can create adverse selection or moral hazard problems.[2] Researchers also tried to study how those information asymmetries can explain the above mentioned low risk premia and high volumes on markets prior to the crisis and followed by the sudden market dry-up. Some of these models resort to irrationality (e.g. Shleifer and Vishny (2010) or Gennaioli et al. (2013)). This paper can reproduce the mentioned phenomena in a purely rational expectations framework by a varying degree of asymmetric information about the quality of the securitized assets.

To study securitization with its problematic aspects over the business cycle, I build a dynamic stochastic general equilibrium model of financial intermediation through securitization in an environment with heterogeneous investment opportunities and information frictions. Financial firms with access to investment opportunities need funding, which can be obtained by sale of their older securitized assets and by securitizing the future cash flows from the current investment opportunity. Crucially, I assume that firms other than the original issuers of securitized assets cannot identify the asset quality unless they hold them and are able to observe their cash flows, which have to be informative. Even in this case, such information acquisition is only private. This assumption is motivated by high complexity,[3] limited standardization and a resulting

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[1]ABCP were the assets issued by the Special Investment Vehicles (SIV) to back investment into securitized pools of loans such as MBS.

[2]See e.g. Shin (2009) or Paligorova (2009).

[3]An extreme example were collateralized debt obligations squared (CDO squared). These were

opacity of securitized assets making them hard to price.[4]

Since firms which originate securitized assets have superior information about asset quality, they have incentives to signal this quality to buyers by retaining part of the risk either explicitly (legally enforced) or implicitly (enforced by reputation). I focus on implicit risk retention (or implicit recourse), which was preferred by mortgage issuers due to its regulatory arbitrage potential.[5] Implicit recourse is a non-contractual support to holders of securitized assets enforced in a reputation equilibrium.[6] There exists a large theoretical and empirical literature on implicit recourse, e.g., Gorton and Souleles, 2006, Mason and Rosner, 2007, Higgins and Mason, 2004 or Ordoñez, 2014. Brunnermeier (2009) also documents implicit (reputational) liquidity support.

The first main contribution of this paper is to study the effect of implicit recourse default on the price and liquidity in the resale securitized markets[7] and finally on investment and output. Due to the mentioned opacity, holders of securitized assets find their intrinsic quality only when assets' cash flows are informative. Since such information acquisition is private, this potentially results in the presence of informed sellers, which creates a standard adverse selection problem in the spirit of Akerlof (1970).

The implicit guarantees may prevent private information acquisition about the asset quality. Indeed, I show that there exist pooling equilibria, where both assets of high and low quality are issued, bear the same level of implicit recourse and as a result generate the same cash-flows after accounting for the implicit guarantee, the information about loan quality remains hidden. Neither seller nor buyers are informed about the quality of traded assets, and therefore, there is no adverse selection in the resale market. The price in the resale market is high, which increases the resources of agents with investment opportunities (liquidity sellers), and boosts investment and output in the economy. This equilibrium is similar to the "blissful ignorance" equilibrium introduced

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assets backed by cash flows from other CDOs, which themselves were backed by various asset backed securities (ABS).

[4]Arora et al. (2012) show that, for some derivatives, it may be prohibitively costly to find their intrinsic quality and price them correctly.

[5]There is a widespread view among economists that securitization itself was taking place due to its potential of arbitraging the capital regulation (e.g. Gorton and Pennacchi, 1995; Gorton and Metrick, 2010; Gertler and Kiyotaki, 2010; and Acharya et al., 2013, among many others).

[6]In this model, default on implicit recourse may trigger a punishment in the form of an inability to issue new securitized assets in the future. In reality, such implicit guarantees are not tracked by regulators and do not result in higher capital requirements for originators of securitized assets.

[7]The distinction between the primary market for newly securitized assets and the resale market for older assets is for the sake of keeping the model realistic. It is unlikely that the buyers cannot differentiate those two markets.

in Gorton and Ordoñez (2014), in which both sellers and buyers decide not to produce information about intrinsic collateral value, resulting in the absence of adverse selection and increases in borrowing and consumption. Unlike in Gorton and Ordoñez (2014), the pooling equilibrium in this model is achieved by provision of reputation based implicit recourse and disappears when issuers of securitized assets find it optimal to default on implicit recourse. The default takes place when the economy is hit by a significant productivity dispersion shock. Such a shock lowers the cash-flows from projects backing low quality assets relative to projects backing high quality assets, which makes the provision of implicit recourse for issuers of low quality assets expensive. Following the literature on uncertainty shocks, the cross-sectional dispersion of productivity is countercyclical in this model, see e.g. Bloom (2009) and Bloom et al. (2012). In the Markov state with the highest productivity dispersion (called "deep recession"), default on the implicit recourse makes the cash flows of all assets suddenly informative. Holders of assets identify privately their quality and the adverse selection in resale markets surges. This may even cause partial market shutdowns, when high quality assets stop being sold altogether. The surge in adverse selection depresses asset price, which in turn limits the resources of agents with investment opportunities, and as a result depresses further the investment and the output in the economy. Such findings are in line with the empirical evidence found by Jordà et al. (2013) suggesting that financial crisis recessions are deeper than normal recessions.

The existence of pooling equilibria with reputation based implicit recourse during the boom stage of the business cycle and a sudden increase in adverse selection following a dispersion shock, can explain the mentioned behavior of securitization markets prior and during the recent financial crisis.

The second main contribution of this paper are the implications of the above identified mechanism for the government policy of asset purchases, inspired by the quantitative and credit easing of the Federal Reserve in the USA. I show that when the government introduces an asset purchase policy in the state of the economy with the most severe adverse selection in the resale markets, the negative effects of the adverse selection on the real economy may be eliminated. However, this policy also generates a negative moral hazard effect, which tends to increase ex ante the issuance of low quality assets, but also a positive general equilibrium effect of less restricted financing constraints. The latter counteracts the moral hazard effect.

The paper is the most closely related to the recent literature which incorporates asymmetric information in financial intermediation into general equilibrium models,

Figure 1. Risk premia surged and market volumes plummeted in the summer of 2007
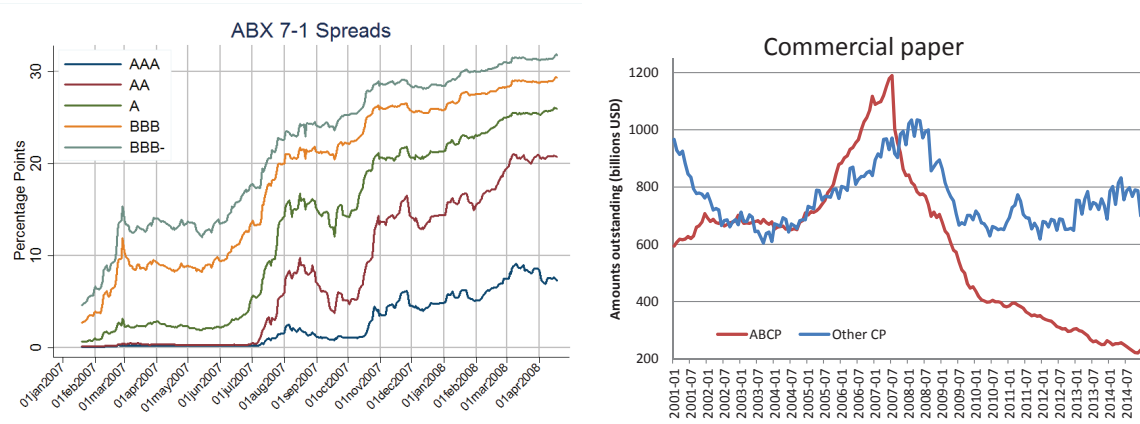
Notes:

The **left panel** is reproduced from Brunnermeier (2009) and shows the ABX 7-1 Spreads (credit default swaps on 20 subprime mortgage securitizations issued in the latter half of 2006) for different tranches. You can observe the dramatic increase in spreads in the summer of 2007. Source of data: LehmanLive.

The **right panel** shows the evolution of amount outstanding of the ABCP compared to other commercial paper over time. You can observe a dramatic drop in amounts outstanding for ABCP in the summer of 2007. Source of data: Board of Governors of the Federal Reserve System (US).

but also to the literature on dispersion shock and on the reputation of financial intermediaries. Similarly to Kurlat (2013), I also find that in an environment with asymmetric information, adverse selection increases in a recession and may even lead to market shutdown. As in Bigio (2015) dispersion shocks are the main reason which increases the adverse selection. But unlike those two papers, the transmission mechanism in this paper incorporates reputation based recourse. This implies an additional amplification mechanism compared to Kurlat (2013), and compared to Bigio (2015) the effect of the dispersion shock is not gradual, but characterized by a jump caused by an economy-wide default on reputation recourse. This paper also shares some results with Ordoñez (2014), who finds that the reputation based financial intermediation is more fragile in a recession. However, unlike Ordoñez (2014), I study the implictations of this fragility for the degree of adverse selection in securitization markets. The closest paper is Kuncl (2015), which also features reputation based recourse in a DSGE model. This paper replicates the results of Kuncl (2015) such as that the depth of the recession is proportional to the length of preceding boom period, during which low quality investments accumulate on financial firms' balance sheets. But it adds results related to default on the implicit recourse and analyses implications for the government policy.

The remainder of the paper is organized in the following way. Section 2 introduces the set-up of the model. Section 3 shows the main properties of the model and the effects of model assumptions analytically in a static framework and then introduces the methodology for the solution of the dynamic model in a Markov regime switching set-up. Finally, the dynamic properties of the model are described based on the solution of the Markov regime switching model and the effect of the government policy of asset purchases are evaluated.

2 Model set-up

The framework of the model is based on more generally on the representative household set-up used in macroeconomic models featuring prominently financial intermediation Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). In this model, financial intermediation is carried out through means of securitization (financial assets backed by future cash flows from a project). Such financial intermediation is subject to information frictions and reputation based implicit recourse is used to overcome those frictions as in Kuncl (2015). But unlike in Kuncl (2015), the recourse is provided for the whole lifetime of the asset and the model features equilibrium defaults on the recourse. The model focuses on how the provision of infinite-horizon reputation-based implicit guarantees interacts with the adverse selection problem in the resale markets.

2.1 Physical set-up

There is a continuum of projects located each on one of a continuum of islands. Each project can produce output using capital as input. The production function has constant returns to scale on the level of the individual project, but decreasing reruns to scale on the aggregate level.[8] As in Kiyotaki and Moore (2012) and Gertler and Kiyotaki (2010), capital is not mobile across islands. Each period, an i.i.d. shock makes projects on $\pi\mu$ fraction of islands highly productive, projects on $\pi(1-\mu)$ fraction of islands less productive and projects on $1-\pi$ fraction of islands unproductive. The production function for projects with high and low production technology, respectively,

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[8]Kiyotaki and Moore (2012) assume a Cobb-Douglas production function with capital and labor as inputs. Due to competitive labor markets, they find that returns to capital are decreasing on aggregate level, while constant on the level of individual firm. For simplicity, this result is taken here as an assumption.

is the following:

$$y_t^h = r_t^h k_t = A_t \Delta_t^h K_t^\alpha k_t,$$
$$y_t^l = r_t^l k_t = A_t \Delta_t^l K_t^\alpha k_t,$$

where y_t^i is the amount of output of a project with productivity $i \in \{h, l\}$, A_t is the aggregate level of total factor productivity (TFP), Δ_t^i is the type-specific component of TFP, K_t is the aggregate level of capital used in production and k_t is the level of capital used in this particular project.

Type-specific components of TFP are functions of A_t. In particular, following the evidence from Bloom (2009) and Bloom et al. (2012), the cross-sectional variance of TFP across firms is counter-cyclical. Therefore,

$$\frac{\partial \left(\Delta_t^h - \Delta_t^l \right)}{\partial A_t} < 0. \tag{1}$$

Capital on islands increases with new investment and depreciates over time with a constant depreciation rate $(1 - \lambda)$. Therefore, the law of motion for the aggregate level of capital is:

$$K_{t+1} = X_t + \lambda K_t,$$

where X_t is the aggregate level of investment in period t.

2.2 Household

There is a representative household with a continuum of members and the size normalized to one. Within the household, there is perfect consumption insurance. For simplicity unlike Gertler and Karadi (2011) and Gertler and Kiyotaki (2010), this model abstracts from labor, and therefore, all household members are called financial firms. Financial firms manage all wealth in the economy N_t and distribute dividends to the aggregate household, which are used to finance consumption of all household members.

Formally, the household maximizes the objective function:

$$E_t \sum_{s=0}^{\infty} \beta^i \log \left(C_{t+s} \right),$$

where C_t is the household consumption. The budget constraint for the household is:

$C_t = \Pi_t$, where Π_t are the distributed dividends from financial firms.

Following Gertler and Kiyotaki (2010), financial firms are subject to exogenous exit shock. In particular, with a probability $(1 - \sigma)$ a financial firm exits, and transfers all equity to the household. An exiting firm is replaced by a new firm, which receives limited start-up funds from the household (in particular $\xi / (1 - \sigma)$ fraction of equity of exiting firms such that $\beta > \sigma + \xi$). Therefore the distributed dividends are equal to:

$$\Pi_t = N_t (1 - \sigma - \xi), \tag{2}$$

The assumption on binding exit shocks is convenient for the purpose of this model, which will feature a reputation based implicit recourse implicit recourse.[9] For a reputation equilibrium to exist, a loss of reputation has to lower the value of equity. Therefore, the marginal value of equity should exceed its unitary costs.[10]

Each financial firm is situated on an island and has exclusive access to the projects on this island. Given the investment shock to the productivity described above, the financial firm has either a high quality investment opportunity with probability $\pi\mu$ (subset $\mathcal{H}_t$ of firms), a low quality investment opportunity with probability $\pi (1 - \mu)$ (subset $\mathcal{L}_t$ of firms), or has no access to any new productive projects this period with probability $1 - \pi$ (subset $\mathcal{Z}_t$ of firms). The investment shock creates the need for financial itermediation.

2.2.1 Financial intermediation frictions

Financial intermediation is carried out through trade of securitized assets which give the holder right to future cash flows from a particular project. Such financial intermediation is subject to **two major frictions**:

1. It is hard to discover the intrinsic value of securitized assets, in particular for the buyers of these assets. This may result in asymmetric information in the markets, where the informed party are:

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[9]As I explain later, implicit recourse is enforced by a trigger punishment rule as in Kuncl (2015). When the punishment is applied by buyers, firm which defaulted on its previously provided implicit recourse cannot sell newly issued assets. This is costly to the firm only when liquidating the banks equity is inefficient, i.e., when the value of equity exceed the unitary costs ($E_t \left(\Lambda_{t,t+1} R_{t+1}^N \right) > 1$).

[10]Should the value of equity be optimal, i.e., $E_t \left(\Lambda_{t,t+1} R_{t+1}^N \right) = 1$, then the marginal value of equity would be equal to one. Any firm after losing its reputation would simply be liquidated and there will be no costs of losing reputation.

- Issuers of securitized assets, which arelocated on the island of the financed project and directly finance the investment opportunity. Therefore, ex ante there is **asymmetric information** about the quality of newly securitized assets sold in the primary market.

- Holders of securitized assets who identify their quality when their observed cash flows are informative, i.e., are distinct from cash flows of other types of assets. This may create asymmetric information in the resale markets.

2. Investing firms, which decide to securitize part of their investment, have to keep a **"skin in the game"**, i.e., they have can sell at most θ fraction of the current investment.[11]

The first friction is supposed to model the main criticism of securitization. It is the argument that the asymmetry of information in securitization markets is the main source of the problems with securitized assets. The idea that it is hard to find the intrinsic value of the asset is supposed to model the high complexity of those assets in reality which made their pricing very costly. Also these opaque assets have been traded often on the OTC markets and public information available for their potential buyers was limited. This friction gives rise to asymmetric information in the primary market (i.e., between issuers and first buyers) as in Kuncl (2015) but also in the resale market. The latter is due to the assumption that holder of an asset can privately observe its cash flows, which may lead to an information advantage, and as a result, to adverse selection problems in the resale market.

The second friction when binding makes securitization profitable despite competitive markets and firms value access to securitization markets. Only then provision of implicit guarantees, enforced by a threat of a loss of market access after default, can be provided in equilibrium.

2.2.2 Financial firms' problem

In this section I formally define the problem faced by each of the financial firms.

The return on equity exceeds its unitary costs:

$$E_t \left(\Lambda_{t,t+1} R_{t+1}^N \right) > 1, \tag{3}$$

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[11]For simplicity θ is taken as a parameter. Kuncl (2015) shows that this friction can be endogenized by the existence of a moral hazard problem. Fixing θ does not alter the qualitative results of the paper.

where $\Lambda_{t,t+1} \equiv \beta \frac{C_t}{C_{t+1}}$ is the stochastic discount factor and R_{t+1}^N is the return on firms' equity.[12]

Therefore, as in Gertler and Karadi (2011), each financial firm (indexed by i) maximizes the following value function (its distributed profit function):

$$V_{i,t}\left(n_{i,t}; S_t\right) = \max E_t \sum_{s=0}^{\infty} \left(1 - \sigma\right) \sigma^s \Lambda_{t,t+s} n_{i,t+s},$$

where n_t is the equity of the individual financial firm and S_t is the set of all state variables. They maximize the above by choosing its control variables $\{x_{i,t+s}, \{a_{i,j,t+s}^p\}_j, a_{i,t+s}^s,$ $\{r_{i,t+s,t+s+k}^G\}_{k=0}^{\infty}, \varphi_{i,t+s}, z_{i,t+s}\}_{s=0}^{\infty}$.

In particular, in every period, each financial firm chooses whether and how much to invest in a new investment project $x_{i,t}$ available on this island. I will denote the subset of firms which decide to invest $\mathcal{I}_t$ and the subset of firms which do not invest, i.e., only save, $\mathcal{S}_t$. When firms invest, they choose how much of this investment to securitize and sell to other firms $\left(x_t - a_{i,i,t}^p\right)$ for the price $q_{i,t}^p$. All firms also choose how many securitized projects to buy from the current issuers (indexed by j) $\{a_{j,i,t}^p\}_j$ for prices $\{q_{j,t}^p\}_j$, how many projects to buy on the secondary markets $a_{i,t}^s$ for the price q_t^s and which projects to keep further on their balance sheet (since the firm may privately find information about those projects, these quantities are a_{t+1}^{hG}, a_{t+1}^{lG} and a_{t+1}^{mG} for projects of high, low and unknown quality with implicit recourse, [13] and a_{t+1}^h, a_{t+1}^l, and a_{t+1}^m for projects of high, low and unknown quality without implicit recourse, respectively). They may sell assets issued in previous periods in the resale market for the unique market price q_t^s, which is independent of asset quality due to the asset opacity.

When they sell the securitized part of the current investment, they may decide to provide an implicit recourse, i.e., an implicit guarantee on the minimum cash flows from the project issued by firm i in time t for the remaining infinite lifetime of the asset: $\{r_{i,t,t+k}^G\}_{k=0}^{\infty}$. In case they have provided implicit guarantees in the past, they also decide whether to default on those guarantees or not, $\varphi_{i,t}$.[14] Financial firms may

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[12]Using (2), you can obtain $C_{t+1} = \left(1 - \sigma - \xi\right)\left(\sigma + \xi\right) N_t R_{t+1}^N$ and substituting this into (3), you obtain $E_t\left(\Lambda_{t,t+1} R_{t+1}^N\right) = \frac{\beta}{\sigma+\xi}$, which exceeds one by assumption.

[13]Given the regulatory limitations on implicit recourse, which are discussed in the next paragraph, the relevant recourse which remains hidden from the regulator can take only the value $r_{i,t,t+k}^G = r_{i,t+k}^h \ \forall k \in (0, \infty)$. Alternatively, the recourse may not be provided at all, i.e., $r_{i,t,t+k}^G \leq r_{i,t+k}^l \ \forall k \in (0, \infty)$. That is why I do not have to keep detailed track of levels of previous implicit guarantees and differentiate the assets accordingly.

[14]$\varphi_{i,t}$ takes two values: $\varphi_{i,t} = 0$ in case of default on implicit recourse or $\varphi_{i,t} = 1$ when the recourse

also use the storage technology and keep consumption goods till the next period $z_{i,t+1}$.

Given the above mentioned options of financial firms, their budget constraints it the following:

$$\sum_{j\in\mathcal{I}_t} a^p_{i,j,t+1}q^p_{j,t} + a^s_{i,t+1}q^s_t + a^h_{i,t+1}q^h_t + a^l_{i,t+1}q^l_t + a^{hG}_{i,t}q^{hG}_t$$
$$+a^{lG}_{i,t}q^{lG}_t + a^{mG}_{i,t}q^{mG}_t + x_{i,t}\left(1-q^p_{i,t}\right) + z_{i,t+1} + \pi_{i,t} \;=\; n_{i,t}\ \forall i, \forall t,$$

where $n_{i,t}$ is the firm's equity after repayment of all current obligations but before the redistribution of dividends, which is defined for a firm that decides not to sell its assets:

$$\begin{aligned} n_{i,t} \;=\;\; & z_{i,t} + a^{hG}_{i,t}\left(r^G_t + \lambda q^{hG}_t\right) + a^{lG}_{i,t}\left(r^G_t + \lambda q^{lG}_t\right) + a^{mG}_{i,t}\left(r^G_t + \lambda q^{mG}_t\right) \\ & +a^h_{i,t}\left(r^h_t + \lambda q^h_t\right) + a^l_{i,t}\left(r^l_t + \lambda q^l_t\right) - \varphi_{i,t}cir_{i,t}, \end{aligned}$$

where $cir_{i,t}$ are the current period costs of honoring the issued implicit recourse guarantees and which are related to the the stock of implicit recourse obligations of this particular firm.

Figure 2. Timing of events withing each period
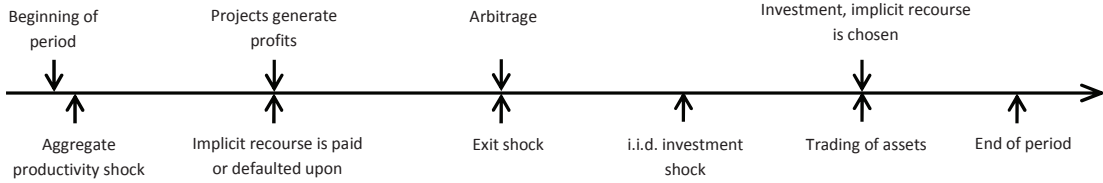
2.3 Implicit recourse

Financial firms selling securitized assets on the primary market can provide the implicit recourse in order to increase the cash flows of sold assets and potentially signal their type. Kuncl (2015) discusses in detail the role of signalling through provision of reputation-based implicit recourse in the form of a promise of minimum cash flows from projects.[15] This implicit recourse is enforced by a threat of punishment in the case of default on the recourse. The punishment does not allow financial firms to sell securitized assets in the future. I focus on equilibria with a trigger strategy punishment. Such a punishment is the most efficient in enforcing the recourse.

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is honored.

[15]Though not modelled here, the advantage of an implicit guarantee as opposed to explicit may be in reality regulatory arbitrage and lower costs of bankruptcy. See e.g. Ordoñez (2014)

Incentive compatible constraints. The incentive-compatible constraints (ICCs) which have to be satisfied at least for some states in the following period $t+1$ for the existence of valuable implicit recourse are:

$$V_{i,t+1}^{ND}\left(n_{i,t+1}^{ND}; \bar{S}_{t+1}\right) \geq V_{i,t+1}^{D}\left(n_{i,t+1}^{DD}; \bar{S}_{t+1}\right) \tag{4}$$

for current issuers of securitised assets $i \in \mathcal{I}_t$, and

$$V_{i,t+1}^{P}\left(n_{i,t+1}; \bar{S}_{t+1}\right) \geq V_{i,t+1}^{NP}\left(n_{i,t+1}; \bar{S}_{t+1}\right). \tag{5}$$

for current buyers of securitized assets $i \in \mathcal{S}_t$.

$V_{i,t+1}^{ND}, V_{i,t+1}^{D}$ are the value functions of the firm i when it has a reputation of not-defaulting on implicit recourse, i.e., does not suffer the punishment, and when it has defaulted already in the past and suffers the punishment, respectively. $V_{i,t+1}^{P}, V_{i,t+1}^{NP}$ are the value functions for the firm i that has a reputation for punishing for defaults on implicit recourse, and for a firm which failed to punish for a default in the past and suffers the negative consequences, respectively. The equity of a firm which has not defaulted on the implicit recourse is $n_{i,t+1}^{ND} = n_{i,t+1} \mid (\varphi_{i,t} = 1)$, and the equity of a firm that used to honor the implicit obligations but has just defaulted for the first time is $n_{i,t+1}^{DD} = n_{i,t+1} \mid (\varphi_{i,t} = 0)$.

When satisfied, the condition (4) implies that the provided implicit recourse is not defaulted upon in the particular future state, given the trigger strategy punishment rule. If the condition is satisfied, the implicit recourse is credible. Similarly the trigger punishment strategy has to be credible, therefore, in the same future state of the world, when (4) is satisfied, (5) has to be satisfied too, i.e., the saving firm observing a default on the implicit recourse has to be better off punishing the investing firm which has defaulted rather than not punishing it.[16]

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[16] Similarly as in Kuncl (2015), I consider the equilibrium in which a firm that has failed to punish will be expected not to punish in the future. Therefore, no firm that would sell it an asset with implicit recourse on the primary market, would honor such implicit obligation toward this firm. Therefore, such firm will have worse conditions on the primary market as in many states of the world they cannot buy an asset that would be for sure free of implicit recourse. As I discuss later, when implicit recourse is being provided in equilibrium, it is provided by firms with access to low quality investment opportunities, who try to mimic cash flows from high quality projects. Sellers of low quality assets would not sell those assets without implicit recourse at a lower price because this would reveal their type. Instead, they would ask the equilibrium price for high quality asset. In Appendix A.4, I claim that this would imply also worse conditions on the secondary market too.

Equilibrium defaults on implicit recourse. In some states of the world, the condition (4) may not be satisfied. Some firms may find it unilaterally beneficial to default on the implicit recourse even when the punishment is expected to be triggered. This would take place in states where honoring the implicit recourse would be too costly, i.e., in particular in a recession where the difference in cash flows between the high and low quality projects is the largest.

It turns out that in states when a sufficiently large fraction of firms default on the implicit recourse (the condition 4 is not satisfied for them), the condition (5) would not hold either. The reason is that the trigger strategy would not be renegotiation proof anymore. The firm which failed to punish, i.e. continues to buy newly issued assets from defaulting firms, may agree on preferential terms of trade with the defaulted firm when such a firm has access to a profitable investment opportunity. Intuitively, when a single infinitesimally small firm defaults on the implicit recourse, the benefits of preferential trade with such a firm are low due to the limited supply of assets by such firm subject to the investment shock. However, when a larger fraction of firms find it optimal to default on implicit recourse, the benefits from preferential trade with them are higher since, due to the law of large numbers the supply of assets is positive in all states.[17] Note that since the punishment is not triggered, all remaining firms will default on the implicit recourse. Therefore the model will feature an economy wide default on implicit recourse without the punishment being triggered. After such an event, the economy may stay in equilibrium without reputation and implicit recourse, or alternatively the economy may move again to a reputation equilibrium where the newly issued assets may carry credible implicit recourse. I will consider the latter case in my infinite horizon model.

Regulatory arbitrage. As already mentioned, one of the main reasons for provision of implicit recourse as opposed to explicit guarantees was the regulatory arbitrage. For this reason this practice was relatively concealed by the issuers. For simplification, I assume that the originators try to conceal implicit guarantee without modeling explicitly the capital requirements regulation which was in this way arbitraged. Therefore, the increased cash flows from the asset should mimic cash flows of some other existing asset, which would make it impossible to distinguish asset with naturally higher cash flows from asset with artificially higher cash flows due to the existence of the implicit

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[17]The credibility of the trigger punishment for default is discussed in greater detail in the Appendix A.4.

support. This assumption introduces some natural limit to the size of the implicit support[18] and simplifies the tractability of the aggregation of infinite horizon implicit guarantees.[19]

The above assumption implies that the level of implicit support is $r^G_{i,t,t+k} = r^h_{i,t+k} \forall k \in (0,\infty)$ or $r^G_{i,t,t+k} \leq r^l_{i,t+k} \forall k \in (0,\infty)$. Note that the latter case is equivalent to the case where implicit recourse is not provided, which is how I will refer to this case. This assumption also limits the number of potential Perfect Bayesian Equilibria compared to Kuncl (2015). I use the Intuitive Criterion by Cho and Kreps (1987), to obtain a unique separating equilibrium as long as a separating equilibrium exists.

Arbitrage prior to the investment shock.   Due to the provision of infinite-horizon implicit recourse, the solution of the model may potentially require keeping track of the distribution of firms' stock of implicit recourse obligations as well as firms' equity. Therefore, to keep the tractability of the model, I make an assumption in the spirit of Gertler and Kiyotaki (2010). In their island economy, to prevent keeping track of the distribution of equity across islands they allow for arbitrage at the beginning of each period. In particular, at the beginning of each period "a fraction of banks on islands where the expected returns are low can move to islands where they are high" (Gertler and Kiyotaki, 2010, p. 13). This arbitrage equalizes ex ante expected rates of return to intermediation.

In this model, a similar arbitrage would imply an equal level of equity as well as an equal stock of provided implicit obligations across islands. More details on the implementation of the arbitrage within the model is in Appendix A.2

2.4   Market clearing conditions

There are two types of goods in the model: consumption goods produced by productive projects and capital goods.

Consumption goods market clears if the consumption goods produced in the current period are all either consumed, converted into capital goods, i.e., invested into

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[18]If the projects would represent loans with delinquency rates differing among loans of different quality, such a natural limit would be zero delinquency.

[19]However, the model is solvable even without this assumption, when the level of implicit guarantee is determined by the strictly binding condition (4) as in Kuncl (2015).

new projects, or stored until the next period:

$$Y_t + Z_t = C_t + X_t + Z_{t+1},$$

where $Y_t = \left(\omega_t r_t^h + (1 - \omega_t) r_t^l \right) K_t$ is the output from all existing projects in the economy and Z_t is the aggregate storage in the economy from period $t - 1$.

Capital goods markets clearing conditions are derived from the optimization of the financial firms in the economy. In equilibrium, firms which are buying various types of assets have to be marginally indifferent among them.

In this paper, we will be interested in the case when both primary as well as secondary (resale) securitization markets are working, which require their expected return to be equal or higher than the return on storage. Similarly, to have new investment being undertaken, the return from taking advantage of the investment opportunity should not be lower than buying assets on the resale markets. Therefore, we obtain

$$
\begin{aligned}
E_t \left[\Lambda_{t,t+1} R_{t+1}^p \right] &= E_t \left[\Lambda_{t,t+1} R_{t+1}^s \right] = E_t \left[\Lambda_{t,t+1} R_{t+1}^{hG} \right] \dots \\
&\geq E_t \left[\Lambda_{t,t+1} R_{t+1}^z \right], \\
&\leq E_t \left[\Lambda_{t,t+1} R_{t+1}^i \right].
\end{aligned}
$$

where R_{t+1}^i is the return from investing, R_{t+1}^p is the return from buying on the primary market, R_{t+1}^s is the return from buying on the resale market and R_{t+1}^z is the return from storage. When the return from storage is equal to the return from buying assets on the primary or secondary markets, there will be a positive level of storage in the economy.[20]

3 Model solution

3.1 Comparative statics

In this section, I derive analytically the behavior of the model and the effects of the above introduced frictions in the steady state. The subsequent sections show the numerical results for the fully dynamic model in the case where all frictions are binding.

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[20] Derivation of market clearing conditions is in the Appendix A.2.

3.1.1 Effect of the "skin in the game" constraint and asymmetric information on the primary market

The basis of the model is similar to Kuncl (2015). When neither of the two frictions in financial intermediation are binding[21] is binding, only high quality projects are being financed and, due to competition, their market price equals the unitary costs of financing $q^h = 1$. Moreover, storage is not used in equilibrium $Z = 0$. However, unlike in Kuncl (2015), due to the binding exit shock, i.e., $\sigma + \xi < \beta$, there is underinvestment in the economy and the return to investment is higher than in the first best case:[22]

$$r^h + \lambda = \frac{1}{\sigma + \xi} > \frac{1}{\beta}.$$

The introduction of a **binding "skin in the game" constraint** (necessity to keep $1 - \theta$ fraction of the new investment on the balance sheet of the issuer) restricts the supply of securitized assets on the primary market, which despite perfect competition drives their price above the unitary investment costs $q^h > 1$. Kuncl (2015) shows in Proposition 1 that the "skin in the game" constraint is binding as long as it exceeds the ratio of the probability of arrival of high quality projects and the fraction of non-depreciated projects

$$1 - \theta > \frac{\pi \mu}{1 - \lambda}.$$

Even lower θ is needed for a positive level of storage in the steady state. Storage is positive in equilibrium iff[23]

$$1 - \theta > \frac{(\sigma + \xi) \pi \mu + 1 - \sigma - \xi}{1 - \lambda} > \frac{\pi \mu}{1 - \lambda}.$$

Similarly, if θ is sufficiently low, even the price of low quality projects can exceed one $q^l \geq 1$ and in this case low quality projects will be financed in the steady state too, even under public information about the quality of projects as suggested by the Proposition 2 in Kuncl (2015).

Introducing **asymmetric information in the primary market** can lead to the

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[21] Recall that the two main frictions are the "skin in the game" and potential asymmetry of information in both primary and secondary markets.

[22]See the Appendix A.5 for the derivation.

[23]This equation holds in the case when the dispersion between TFP of high and low quality projects are large enough so that only high quality projects are financed in equilibrium. Derivations can be found in Appendix A.5.

existence of a pooling equilibrium in which projects of both qualities are being financed, but they are indistinguishable to the buyers. In a pooling equilibrium, the allocation of investment is inefficiently skewed in favor of low quality projects and there is cross-subsidization from high to low quality issuers. A separating equilibrium, in which only high quality assets are being financed, may exist as long as the difference in loan qualities is large enough. In such case, firms with access to low quality investment opportunities prefer to buy high quality projects rather than investing and mimicking firms with access to high quality investment opportunities:

$$R_i \mid buying\, high\, assets \geq R_i \mid mimicking \forall i \in \mathcal{L}_t.$$

This condition is satisfied if the dispersion in TFP between high and low quality projects is large enough. In particular, as derived in Appendix A.6, a separating equilibrium is possible only if the ratio of high-type and low-type TFP satisfies:

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu} \tag{6}$$

in the case where storage technology is not used in the equilibrium, or

$$\frac{A^h}{A^l} \geq \frac{(\sigma + \xi)\pi\mu + 1 - \sigma - \xi}{(\sigma + \xi)\pi\mu} \tag{7}$$

in the case with positive level of storage in equilibrium. Note that when the economy is more constrained, achieving the separating equilibrium would require a larger dispersion in TFP. The RHS of (6) increases with lower π, μ, θ or lower λ, which constrain the supply of securitized assets more than the demand for those assets, and therefore, increase the return and prices of both high and low quality projects, thus making pooling equilibrium more likely. Similarly, the RHS of (7) increases with lower π, μ, σ or lower ξ. Other parameters in this case influence the size of the storage rather than the investment into low quality assets.

3.1.2 Reputation equilibria with the implicit recourse

The inefficiencies related to the existence of asymmetric information in the primary market can be alleviated by signalling through provision of the implicit recourse. This result is similar to Kuncl (2015) despite non-trivial differences in the provision of implicit recourse. Similarly to Kuncl (2015) implicit recourse is enforced in a reputation

equilibrium, in which conditions (4) and (5) have to be satisfied. The main difference is that the implicit recourse is provided for the whole lifetime of the asset, i.e., it is an infinite horizon recourse. The second difference is the introduction of limits to the size of the implicit recourse. Those are motivated by the fact that in reality regulators try to detect and limit the implicit recourse because they consider it as a means of regulatory arbitrage. To conceal the provision of implicit recourse, it is possible only to improve the cash flows of the project to the level of another existing asset. In this model, this means that the only implicit recourse, which has the potential to affect the equilibrium, guarantees cash flows on the level of a high quality asset: $r_{i,t,t+k}^{G} = r_{i,t+k}^{G} = r_{i,t+k}^{h} \ \forall k \in (0, \infty)$.

The provision of implicit recourse, which is more costly for the issuers of low quality assets makes the separating equilibrium more likely. In particular, a separating equilibrium exists iff

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)(1 + B)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu + B(1 - \pi\mu)(1 - \lambda)(1 - \theta)} \tag{8}$$

in the case without usage of storage technology and

$$\frac{A^h}{A^l} \geq \frac{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)(1 + B)}{(\sigma + \xi)\pi\mu + B((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)} \tag{9}$$

in the case with usage of storage technology. The RHS of those conditions are lower than in conditions (6) and (7), respectively.[24] Therefore, as a result of the introduction of the implicit recourse, a larger set of cross-sectional dispersion in TFP is consistent with a separating equilibrium.

3.1.3 Asymmetric information in the resale market

So far, we have considered the asymmetry of information in the primary market, i.e., between the originators of securitized assets and buyers of these assets. The results of these frictions have been similar to those in Kuncl (2015) despite several differences. However, the focus of this paper is the asymmetry of information in the resale market.

In this section, I describe the effects of the information frictions between traders in the resale market. I have assumed that only holders of the asset may privately observe its quality provided that its cash flow is informative. This assumption may lead to

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[24]For proof, see the Appendix A.7.

asymmetric information between sellers and buyers on the resale market, which causes a typical adverse selection. The new results in this paper come from the interaction of the adverse selection in the resale market with the switching between pooling and separating equilibria over the business cycle in the primary market, and with the provision of implicit recourse.

Case without provision of implicit guarantees. To demonstrate the effect of switching between the pooling and separating equilibria on the adverse selection problem, let's consider first the case without the provision of implicit guarantees.

The assumption of asymmetric information in resale markets has the following impact on the model behavior. When an asset is re-sold, there is a unique price that is independent of the quality of this asset q_t^s, which depends on the share of high quality assets sold in the market.[25] In every period, there are liquidity and informed sellers in the market. Firms with access to profitable investment opportunities may decide to sell even high quality assets to finance the costs of the investment. I will refer to these sellers as liquidity sellers. In every period, all holders of the assets observe the cash flows from the projects on their balance sheet. Without the provision of the implicit recourse, they will be able to privately identify which assets are of high quality (value q_t^h) and which of low quality (value q_t^l). Due to the presence of liquidity sellers selling both high and low quality assets, the market price exceeds teh value of low quality assets $q_t^s > q_t^l$. Therefore, when a low quality asset is privately identified it is sold in the resale market. These sellers are called informed sellers.

Therefore, when the binding "skin in the game" constraint makes investment profitable such that all investing firms sell all of their asset holdings including high quality assets to boost their investment, the share of high quality assets in the resale market is:

$$f_t^h = \frac{\pi \mu \omega_t}{\pi \mu + (1 - \pi \mu)(1 - \omega_t)} \tag{10}$$

in the case of a separating equilibrium, where $(1 - \pi \mu)(1 - \omega_t)(\sigma + \xi) K_t$ are the low quality assets sold by informed traders and $\pi \mu (\sigma + \xi) K_t$ are the assets sold by liquidity traders. In a pooling equilibrium this condition becomes

$$f_t^h = \frac{\pi \omega_t}{\pi + (1 - \pi)(1 - \omega_t)}. \tag{11}$$

---

[25]See Appendix A.8 for details.

The steady state which is a separating equilibrium is characterized by $\omega = 1$ and by the fact that only high quality assets are being traded on the resale markets. Therefore, $f^h = 1$ and $q^s = q^h$. However, if there is a pooling equilibrium in the steady state, then $\omega = \mu$,

$$f^h = \frac{\pi \mu}{\pi + (1 - \pi)(1 - \mu)} < 1,$$

and $q^l < q^s < q^h$. Therefore, due to the adverse selection, liquidity traders sell high quality assets for too low a price and informed sellers sell low quality assets for an overvalued price. There is inefficient cross-subsidization of informed traders by liquidity traders, which reduces the investment and output in the economy.

If, due to the adverse selection, the price of assets on the resale market drops low enough, even firms which sell assets for liquidity reasons will cease selling high quality assets. The price is so low that the return from taking advantage of the investment opportunity would not compensate for the cost of selling a valuable asset at a low market price. In a deterministic steady state, this situation takes place if:

$$V_i \left(keeping\, high\, projects \right) \geq V_i \left(selling\, high\, projects\, and\, investing \right) \ \forall i \in \mathcal{H}.$$

As shown in Appendix A.8, this condition implies that the share of high quality assets traded on the resale market has to be low enough to satisfy:

$$f^h \leq \frac{1 - \theta \mu q^h - (1 - \theta \mu) q^l}{(1 - \theta)(q^h - q^l)}. \tag{12}$$

This condition is satisfied when the dispersion in qualities is large enough (i.e., for sufficiently large difference $q^h - q^l$). Note that there will never be a complete market shutdown since low quality assets would still be sold at a fair price q^l_t. But the volume of sales would diminish due to the absence of high quality assets in the market, and the level of overall investment in the economy would also be significantly reduced.[26]

The dynamic implications are demonstrated in greater detail in the next sections, but the basic intuition can be shown here based on the above derivations. The prices in the resale market q^s_t depend positively on the share of high quality assets sold on the market f^h_t and negatively on the dispersion of qualities between the two assets. The share of high quality assets f^h_t in turn depends positively on the share of high

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[26]In the dynamic solution of the model, I do not have partial market shutdowns, since such non-linearities and their duration are hard to endogenously establish in the model, however, I show the varying degree of adverse selection.

quality assets in the economy ω_t as shown in (10) and (11). Therefore, since recessions are characterized by a larger dispersion in qualities, intuitively the adverse selection is more important in a recession than in a boom. Furthermore, since low dispersion between the qualities in the boom leads to the occurrence of pooling equilibria in the primary market, the longer the boom period is, which precedes the recession, the larger is the share of low quality loans in the market and the more acute the adverse selection issue becomes. If adverse selection is strong enough, securitized loans of high quality cease being traded in the resale markets altogether, which further deepens the recession.

Case with provision of implicit recourse. The provision of infinite horizon implicit recourse influences the problem of adverse selection in resale markets in two ways.

The **first effect** of implicit recourse provision is on **the lower effective difference between the value of high quality assets and low quality assets with implicit recourse**. Since low quality assets with implicit recourse will have the same cash flows as high quality assets, the resale market price is much less negatively influenced by the presence of the low quality assets with implicit recourse. Indeed, it is the presence of low quality assets without implicit recourse which significantly negatively influences the resale market price q^s.[27] Therefore, as long as all low quality assets bear implicit recourse making their cash flows equal to high quality assets, the resale market works relatively well. However, after a potential default on implicit recourse, low quality assets with low cash flows will appear in the resale market and negatively influence its price. This becomes especially pronounced when such a default is widespread in the economy. In the next sections I will show that this is the case after a large dispersion shock (in a deep recession).

The **second effect** of implicit recourse provision is related to its effect on **the degree of asymmetric information in the resale market**. I have assumed that implicit recourse is costly to detect, and therefore, holders of an asset may find its quality only based on the cash flows it generates. As long as the implicit recourse is being provided, holders cannot distinguish between high quality assets and low quality assets with implicit recourse. However, when implicit recourse is being defaulted upon, low quality assets are easily privately identified and a large quantity of informed sellers appear in the resale market. As I show in the next section, the default on implicit recourse is limited to the exiting firms in boom times or mild recessions, but they are

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[27]Note that even in the steady state, there are low quality assets without the implicit recourse. This is due to the exit shock. Exiting firms of course do not provide implicit recourse in the future periods.

widespread in deep recessions, when the dispersion in qualities becomes too large to continue providing implicit recourse. This implies that in booms and mild recessions, the problem of asymmetric information, and therefore, of adverse selection in resale markets is marginal, but becomes very severe in a deep recession.

I show in Appendix A.8 that the prices on the resale market q_t^s are negatively affected by the fact that, in the following period, the share $f_{t+1}^{NIR}\left(1-f_t^h\right)$ of assets sold in the resale market will generate only low cash flows, where f_{t+1}^{NIR} is the share of low quality assets without implicit recourse (out of all low quality assets), and the share of high quality assets is given by

$$f_t^h = \frac{\pi \omega_t}{\pi + f_t^{NIR}\left(1-\pi\right)\left(1-\omega_t\right)}. \tag{13}$$

Liquidity traders sell π fraction of capital, out of which ω_t is the share of high quality assets, and informed traders sell $f_t^{NIR}\left(1-\pi\right)\left(1-\omega_t\right)$ fraction of capital on the resale market.[28]

In this case with implicit recourse, the share of high quality assets f^h again positively affects the resale market price q^s. Moreover, the market price is negatively affected by the share of low quality assets without implicit recourse f_t^{NIR}. A high f_t^{NIR} implies low cash flows from assets bought in the resale market and a higher share of informed traders in the resale markets. The latter lowers the share of high quality assets sold in the market f_t^h.

3.2   Methodology for solution of the dynamic model

This section presents the methodology which is used to solve the fully dynamic model. The model is too complex to be computed by global numerical approximation methods as in Kuncl (2015). In particular, it contains four state variables $\left(A_t, K_t, \omega_t, f_t^D\right)$[29], which make the iteration on the grid of state variables challenging. Therefore, I use a perturbation method, i.e., I find the linear approximations of the policy functions

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[28]Note that I assume that, between periods, any potential information about the asset quality is lost and has to be learned again. This assumption is not crucial for the results but simplifies the solution and rules away the adverse selection by the original issuers of low quality assets who might decide to hold the skin in the game only for one period. In reality, the skin in the game is held longer, but for tractability, I do not want to make such a restriction and I rather assume the loss of information between periods.

[29]f_t^D is the share of low quality assets without the implicit recourse at the end of the period which is more convenient state variable in the recursive formulation of the model than f_t^{NIR}. The relation between f_t^D and f_t^{NIR} is explained in detail in the appendix B.1.

around the steady state which determine the laws of motion for the model variables.

The equilibrium conditions of the model are very different for various combinations of state variables. Standard perturbation methods cannot capture this non-linearity. Therefore, to solve this model, I use perturbation method for Markov-switching DSGE models using the methodology introduced by Foerster et al. (2013).

Foerster et al. (2013) propose an algorithm which can provide first- and second-order approximation for policy functions for Markov-switching rational expectations models where some parameters follow a discrete Markov chain process indexed by s_t. The Markov chain has a state-independent transition matrix $\mathcal{P} = (p_{s,s'})$.

The model equilibrium conditions can be written in a general form as

$$E_t f \left(y_{t+1}, y_t, x_{t+1}, x_t, \chi_{t+1}, \chi_t \right) = 0_{n_x + n_y}, \tag{14}$$

where y_t is an $n_y \times 1$ vector of non-predetermined (control) variables, x_t is an $n_x \times 1$ vector of predetermined (state) variables, which are known already at time $t - 1$, and χ_t is the vector of Markov switching parameters. In our case, there are 4 state variables $x_t = \left(A_t, K_t, \omega, f_t^D \right)$, i.e., $n_x = 4$. Markov-switching parameters χ_t can influence the values of the steady state. To compute a unique steady state Foerster et al. (2013) propose to use the mean of parameters' ergodic distribution across Markov regimes $\bar{\chi}_t = \sum_s p_s \chi_s$, where p_s is the unconditional probability of occurrence of Markov regime s ($s \in \{1, \ldots, n_s\}$).

The solution of the recursive model (14) is

$$
\begin{aligned}
y_t &= g \left(x_t, \psi, s_t \right), \\
y_{t+1} &= g \left(x_{t+1}, \psi, s_{t+1} \right), \\
x_{t+1} &= h \left(x_t, \psi, s_t \right),
\end{aligned}
$$

where ψ is the perturbation parameter. We do not know the explicit functional form for g and h and therefore, we do a first-order Taylor expansion around the steady state. The first order approximations g^{first} and h^{first} are

$$
\begin{aligned}
g^{first} \left(x_t, \psi, s_t \right) - y_{ss} &= Dg_{ss} \left(s_t \right) \boldsymbol{S_t}, \\
h^{first} \left(x_t, \psi, s_t \right) - x_{ss} &= Dh_{ss} \left(s_t \right) \boldsymbol{S_t},
\end{aligned}
$$

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where $\boldsymbol{S_t} = \left[(x_t - x_{ss})^T \ \psi \right]^T$ and $\{Dg_{ss}(s_t), Dh_{ss}(s_t)\}_{s=1}^{n_s}$ are the unknown matrices. Foerster et al. (2013) use the method of successive differentiation to find these unknown matrices. They show that this problem can be reduced to finding a solution to a system of quadratic equations. Finally, Foerster et al. (2013) check the stability of the solution using the concept of mean square stability (MSS) defined in Costa et al. (2005).

The algorithm works only with constant transition probabilities, while our model predicts that the change between different regimes endogenously depends on the four state variables $\left(A_t, K_t, \omega_t, f_t^D\right)$. Only the level of TFP (A_t) is exogenous in this model and K_t, ω_t, f_t^D are endogenous variables. It is the A_t together with the dispersion between TFP of high and low quality projects, which is related to A_t by equation (1), that is the main determinant of the switch between a pooling equilibrium and a separating equilibrium and a default on implicit guarantees. Therefore, I construct a Markov process for A_t and the related Δ_t^h, Δ_t^l such that for a subset of endogenous state variables K_t, ω_t, f_t^D around the steady state the endogenous conditions for the existence of a separating or pooling equilibrium and for default or non-default on implicit support predict the same type of equilibrium for the particular Markov regime. This reconciles to some extent the need for constant transition probabilities in the used solution algorithm and the endogenous conditions for the change in the above mentioned regimes.

The exogenously switching regimes, which satisfy the endogenous conditions, have the following properties for this subset of state variables:

Regime 1 - Expansion: high aggregate TFP $(A_1 = A_H)$ and lowest dispersion in type specific TFP$\left(\Delta_1^h - \Delta_1^l\right)$ make this a pooling equilibrium;

Regime 2 - Mild Recession: low aggregate TFP $(A_2 = A_L)$ and higher dispersion of type specific TFP$\left(\Delta_2^h - \Delta_2^l > \Delta_1^h - \Delta_1^l\right)$ is sufficient to make this a separating equilibrium but implicit recourse is still being honored; and

Regime 3 - Deep Recession: the low level of aggregate TFP $(A_3 = A_L)$ and the highest dispersion of type specific TFP $\left(\Delta_3^h - \Delta_3^l > \Delta_2^h - \Delta_2^l\right)$ not only make this a separating equilibrium, but also all firms, upon arrival to this regime, find it optimal to default on their outstanding implicit recourse obligations.

Note that the dispersion shock is necessary to achieve the difference in the types of equilibria. The change in the TFP level only amplifies the effects induced by the dispersion shock.

I also assume some particular properties of the transition matrix $\mathcal{P}$. First, I assume that the economy typically switches between the expansion and mild recession, while

rarely the expansion is followed by a deep recession so $p_{1,2} \gg p_{1,3}$ and $p_{2,3} = 0$. Since the defaults on implicit guarantees take place only upon entry to Regime 3, and therefore, the equilibrium conditions would be different for the first period in Regime 3 and compared to the subsequent periods, I assume that $p_{3,3} = 0$.

3.3   Dynamic properties of the model

In this section, I show the results of the dynamic fully stochastic model with the above introduced three Markov regimes to illustrate the dynamic implications of the model with the focus on the effects of the adverse selection on the resale markets.

I then introduce a government with a policy of asset purchases in a deep recession state and I show that such policy limits the negative effects of the adverse selection on the real economy.

3.3.1   Benchmark case

Parametrization of the model.   In this section, I focus on the case when both financial intermediation frictions introduced in section 2.2.1 bind. As demonstrated in the preceding steady state derivations, this restricts some of the parameters. Furthermore, to reconcile the methodology by Foerster et al. (2013), which requires exogenous transition probabilities between Markov regimes, with the endogenous model conditions for a significant subset of state variables, I need significant differences in some of the parameters across the regimes. Following Kiyotaki and Moore (2012), I set $\alpha = 0.4$ and $\beta = 0.99$. The persistence parameter for the productivity process is set to $p_{1,1} = p_{2,2} = p_{3,2} = 0.86$.[30] I assume that deep recession can only follow an expansion period, i.e., $p_{2,3} = 0$. The probability of a deep recession is set to be very low compared to mild recession: $p_{1,3} = 0.005$ and $p_{1,2} = 1 - 0.86 - 0.005$. The deep recession is characterized by the same level of TFP as Regime 2 (A_L) but by higher dispersion in type specific components of TFP. The ratio of aggregate components of TFP is $A_H/A_L = 1.05$ and the ratios of type specific TFP are $\Delta_1^l/\Delta_1^h = 1$, $\Delta_2^l/\Delta_2^h = 0.65$ and $\Delta_3^l/\Delta_3^h = 0.6$. The depreciation rate $1 - \lambda$ is set to 0.18, which is supposed to match the Weighted Average Life (WAL) of securitized assets, which is reported to be on average 5.6 years by Efing and Hau (2013) (p.11). The probability of firms' survival

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[30]This corresponds to an auto-correlation of TFP at a quarterly frequency of 0.95. Note that I have assumed that $p_{3,3} = 0$. Therefore, by persistence in the case of Regime 3, I mean the persistence of the recession (i.e. either Regime 2 or 3).

Figure 3. Economy switches to a pooling equilibrium in boom
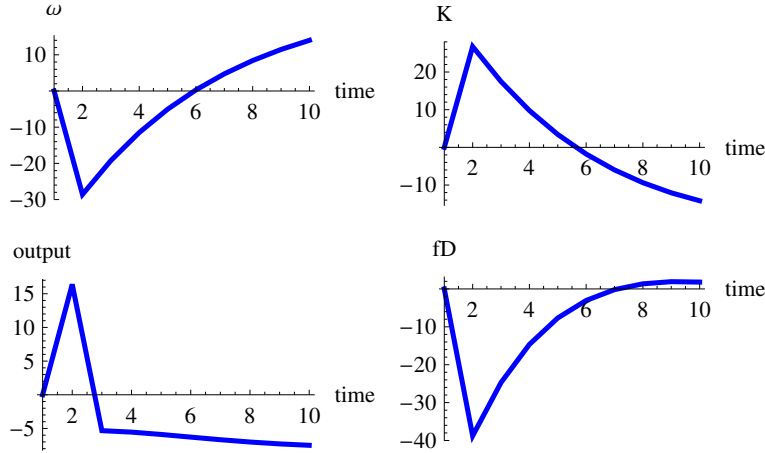
Note: Impulse responses shows the percentage deviations of endogenous variables from their steady state level for an economy which moves for one period to the Expansion Regime and then to Mild Recession.

$\sigma = 0.979$ is set such that the ratio of storage to capital in the steady state is 6% which is comparable to the level calibrated in Kiyotaki and Moore (2012). Parameters $\pi = 0.1$ and $\theta = 0.37$ are set such that the endogenous conditions for pooling, separation and default fit the properties of Markov regimes for a subset of state variables around the steady state.

Impulse responses. The switching between the pooling in the expansion (Regime 1) and the separating equilibrium on the primary market in recession (Regime 2 and 3) is the property shared with Kuncl (2015). Therefore, the main results of Kuncl (2015) are reproduced here. In particular, the longer the economy stays in the boom, the higher will be the share of the low quality assets accumulated on its balance sheet and the deeper will be the subsequent downturn. Figure 3 shows the evolution of endogenous variables for an economy which moves to the expansion (Regime 1) for one period and then to a mild recession (Regime 2). First, due to higher productivity of both high and especially low quality projects, investment, capital and output increase dramatically. Due to lower dispersion in qualities, the economy moves to the pooling equilibrium, therefore the share of high quality assets ω decreases. But the subsequent downturn is deeper due to the accumulation of low quality assets on financial firms' balance sheets.

The main focus of this paper is the effect of asymmetric information on the resale

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markets over the business cycle. Section 3.3 explains that as long as the implicit recourse is provided, the problem of adverse selection in the resale market is limited. This is due to two reasons. When implicit recourse is provided, the cash flows from low quality assets are high. Moreover, it is harder to identify low quality assets and therefore, there are fewer informed sellers on the resale markets. Those positive effects suddenly disappear when the implicit recourse is defaulted upon. This takes place in Regime 3. Figure 4 shows the effect of defaults on implicit recourse. It compares two cases of the economies, both moving from the steady state to the Deep Recession (Regime 3) for one period and then back to the steady state. In the first case (red full curves), the optimizing firms choose to default on the implicit recourse. In the second case (dashed blue curves), the economy is affected by the same shocks, but as a surprise, I do not allow firms to default on the implicit recourse, even though otherwise they would choose to default. Therefore, the difference between the two cases is given by the default on implicit recourse. In the case where default is allowed, all firms default and the share of low quality assets without implicit recourse increases to 100% $\left(f_t^{NIR} = 1 \right)$. The market price in the resale market q_t^s drops due to a severe adverse selection problem while in the case of no default the price on the resale market slightly increases, which is related among others to higher ω_t. Indeed, the economy switched to the separating equilibrium, and therefore, one positive development in the economy is that new low quality assets are not being issued. In the case of default, a low resale market price reduces the resources that investing firms can obtain for selling their assets. Adverse selection causes an outflow of resources from liquidity sellers (investors) to informed sellers. This reduces the investment and the level of capital in the economy drops further. Due to a low supply of new securitized assets, investing firms decide to store more resources rather then spend them on acquisition of securitized assets. All those effects combined have a negative effect on the output of the economy. For the sake of clarity, the Figure 5 depicts the difference in the model variables impulse responses between these two cases. It is clear that due to the default on the implicit recourse and the implied adverse selection problem the resale market price is depressed, which reduces the level of capital and output, but increases the level of storage. Note that these effects are highly persistent.

Figure 4. Effect of defaults on implicit recourse on adverse selection
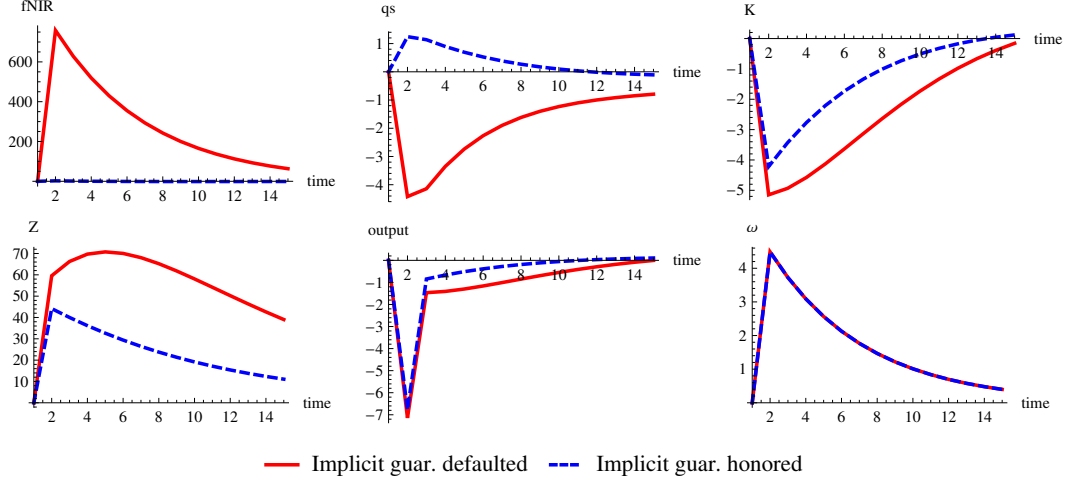
Implicit guar. defaulted   Implicit guar. honored

Note: Impulse responses shows the percentage deviations of endogenous variables from their steady state level for an economy which moves for one period to the Deep Recession Regime and then moves back to the steady state. The red full line shows the case when optimizing firms default on the implicit recourse and the blue dashed line shows the case when, by surprize, such defaults cannot take place.

Figure 5. Effect of defaults on implicit recourse on adverse selection (cont.)
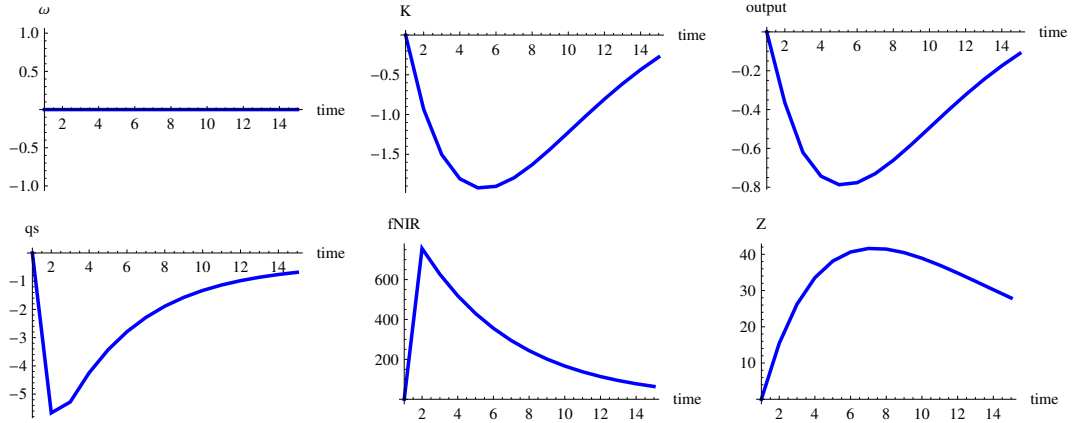
Note: These impulse responses show the difference between the case with defaults and without defaults on implicit recourse from the previous Figure 4. The difference is reported in percentages relative to the steady state level.

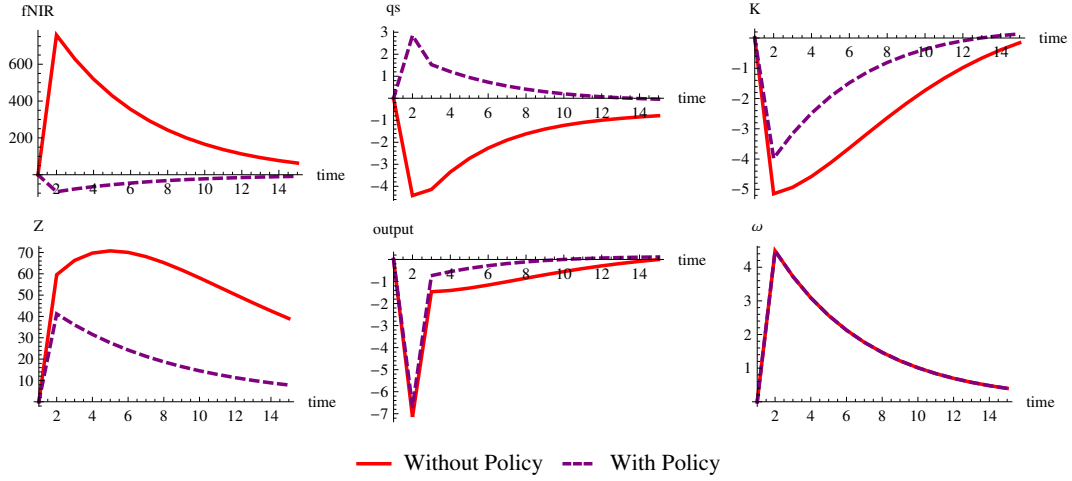Figure 6. Effect of government asset purchase policy

Note: Impulse responses shows the percentage deviations of endogenous variables from their steady state level for an economy which moves for one period to the Deep Recession Regime and then moves back to the steady state. The red full line shows the case when optimizing firms default on the implicit recourse and the purple dashed line shows the case when, in the Deep Recession Regime government policy of asset purchases is introduced.

3.3.2 Government policy of asset purchases

The asymmetry of information creates high inefficiency due to low resale price for assets of liquidity sellers which restricts the investment and output in the economy. A government policy in the form of asset purchases introduced in the Deep Recession Regime (Regime 3), where as a result of defaults on implicit recourse the adverse selection is the most accute, limits the negative effect of the adverse selection on the real economy.

Introducing government policy. In this extension I consider a policy of asset purchases which is motivated by the quantitative and credit easing by the Federal Reserve in the United States. I introduce a new agent, government, in the model. Government may swap securitized assets sold in the resale market for government bonds, while the value for the government bonds may be higher than the market value of the securitized assets. The cost of this policy is charged to financial firms in a form of lump-sum taxes. I introduce this policy in the Deep Recession Regime and show that this policy limits adverse selection effects. I also explore the potential moral hazard effects of this policy.

For simplicity I assume several properties of the government buying scheme, that do

not influence the main qualitative result, but minimize the number of state variables. When teh asset purchase program is activated in the Deep Recession Regime, any financial firm in the economy may decide this period to swap its securitized assets for government bonds promising to pay next period r^B_{t+1}. I assume that the government doesn't have an information advantage over market participants, therefore it cannot restrict the pool of eligible assets to high-quality assets.[31] For simplicity, I assume that this bond is a one-period bond, but once a particular asset is in the asset purchase program, it can be swapped any following period $t + s$ for a new government bond with the promise to pay r^B_{t+s+1}. But no new assets can enter in the asset purchase program unless the economy returns to the Deep Recession Regime. I also assume that the government credibly commits to bind the bond returns to the conditions in the economy, in particular in the Deep Recession Regime, it will commit to $r^B_{t+1} = E r^h_{t+1}$ and the following periods it will target returns s.t. $q^B_{t+s+1} = q^s_{t+s+1}$, where the q^s_{t+s+1} is the price on the secondary market conditional on all low quality assets from the asset purchase program remaining on the government balance sheets. This ensures that high quality assets that don't need to be sold for liquidity reasons remain on firms' balance sheets, low quality assets with defaulted implicit recourse remain on government balance sheets and do not depress the market price and high quality assets sold to the program for liquidity reasons may exit the program when liquidity reasons disappear.

Therefore, the model remains recursive and I just have to introduce one new state variable, which is the fraction of government bonds as a share of total capital f^B_t.[32] As mentioned earlier, I guess and verify that once the government introduces the program in the Deep Recession Regime, all agents with low quality assets will find it optimal to convert all of their holdings of securitized assets to government bonds. Firms that need to sell high quality assets will convert those assets to bonds only during the program introduction in the Deep Recession Regime. But in the following period, they would prefer to exit the program, if they have no investment opportunities. If they have an investment opportunity, they would be indifferent between staying in the program or selling the assets in the resale market. For simplicity but without loss of generality, I focus on the equilibrium, where this small fraction of firms sell the assets in the resale

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[31]If government had a better screening technology, they would still alleviate the adverse selection problem as holders of high quality assets would be able to sell them at a better price. This would have positive effect on the real economy.

[32]The conversion rate is one unit of a securitized asset to one unit of bond.

markets. The law of motion for bonds as a share of capital is therefore:

$$f_t^B = \left((1 - \chi_{D,t}) f_{t-1}^B + \chi_{D,t} (1 - \omega_t) \right) \frac{\lambda K_t}{\lambda K_t + X_t}.$$

The effect of the introduction of the government policy in the model on the equilibrium conditions is in the Appendix B.3.

Policy effects on adverse selection. Figure 6 shows impulse responses in a situation when the economy moves from the zero-probability steady state for one period to the deep recession before returning to the steady state. Two cases are being compared: the case with the above described policy of government asset purchases and the case without such policy. You can see that thanks to the government policy the share of low quality assets without implicit recourse in the resale markets f_t^{NIR} drops from the steady state level to zero as all low quality assets are transferred to the government balance sheet. Compared to this, in the case without the government policy, the resale market is plagued by the low quality assets due to adverse selection, which can be seen in the sharp increase in f_t^{NIR}. As a result, the price in the resale markets is significantly higher (even above the steady state level) in the case with the government policy. This reduces the effect of the negative productivity shock on investment, capital level and output. While in both cases the output is depressed due to the negative productivity shock, output stays depressed more and longer in the case without the government policy.

Potential moral hazard problems. Higher resale market price induced by the government policy may have indirect effects on the proportion of low quality assets issued over the business cycle. To evaluate this, I have to change the parametrization of the model,[33] so that in the Mild Recession Regime, there is also a pooling equilibrium. Unlike in the Expansion Regime, firms with access to low-quality investment opportunities in the Mild Recession Regime do not invest all their non-consumed resources in low quality investment opportunities. But only ψ_t fraction of their non-consumed resources are spent on the new low quality investment opportunities and the remainder is spent on acquisition of other assets.

I investigate the effects of introduction of government policy of asset purchase on

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[33]In particular, I increase Δ_2^l/Δ_2^h from 0.65 to 0.67 to obtain a pooling equilibrium in Mild Recession Regime and reduce π from 0.1 to 0.098 to keep storage positive.

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share of the high quality assets ω in the steady state. I find two opposing effects:

1. **Negative moral hazard effect:** Higher resale market price in the Deep Recession Regime tends to increase both prices of high and low quality assets. The positive effect is stronger for low quality assets because low quality assets are all exchanged for government bonds in the Deep Recession Regime. Therefore, the governemnt policy makes the relative difference between high and low quality assets smaller, which contributes to higher issuance of low quality assets in the Mild Recession Regime (higher ψ_t).

2. **Positive general equilibrium effect of less restricted financing constraints:** Higher resale market price in the Deep Recession Regime increases the amount of resources available for companies with high quality investment opportunities. This results in higher level of steady state capital and due to decreasing returns to lower returns from investment and securitization. This effect drives the less efficient firms with access to low quality investment opportunities out of investing and results in lower issuance of low quality assets in the Mild Recession Regime (lower ψ_t).

In the parametrization of the model, the second effect is stronger, so the policy of asset purchases does not suffer from a net negative moral hazard effects.

4   Conclusion

This paper can replicate the great boom of the market for securitized products in the period prior to the recent financial crisis and the following collapse of this market characterized by low volumes and high spreads. In a theoretical model, I propose a mechanism within the rational expectations framework which is based on information frictions and asymmetries.

I model the securitization process with its peculiarities such as the asymmetry of information and the provision of implicit recourse by originators of securitized assets. This paper introduces information frictions in the resale market. These may lead to information asymmetries between the sellers and buyers of securitized assets on this market and to the related adverse selection problem. The first contribution of this paper is the study of the interaction of the severity of the adverse selection problem with the provision of the infinite horizon implicit recourse.

The model shows that, thanks to the provision of the implicit recourse, the adverse selection is contained in boom periods and mild recessions. This is due to low dispersion in cash flows generated by the securitized assets supported with the implicit recourse. Moreover, due to the provided implicit recourse, it is harder to find the intrinsic quality of the assets, which limits the number of informed traders in the resale markets. The model also predicts a sudden dramatic increase in adverse selection after a larger dispersion shock, which lowers the cash flows generated by low quality assets relative to high quality assets. This makes the provision of implicit recourse too costly and there is a widespread default on these reputation based guarantees. As a result, the effective dispersion in cash flows generated by different types of securitized assets increases dramatically and the proportion of informed traders on the market also increases. Both effects exacerbate the negative effects of the adverse selection. The price of the assets sold in the resale markets, the investment as well as the output of the economy are persistently depressed.

The second contribution of the paper is the analysis of the government policy of asset purchases. The model results show that such government policy may limit the negative effects of adverse selection on the real economy. However, this policy also generates two side-effects: a negative moral hazard effect and a positive general equilibrium effect of less restricted financing constraints. The latter side-effect counteracts the negative moral hazard effect.

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Appendices

A   Model solution and Comparative statics

A.1   Aggregate stock of implicit recourse

When there is no separating equilibrium, we obtain a single pooling equilibrium, in which both firms with access to high and low quality investment opportunities provide the same level of implicit recourse, $r^G_{i,s,t+k} = r^G_{i,t+k} = r^h_{i,t+k} \, \forall s \, \forall k \in (0, \infty)$. In this case, the aggregate costs of providing implicit recourse[34] issued in period t is

$$
\begin{aligned}
IR_t &= \sum_{i \in \mathcal{L}} x_{i,t} E_t \left(\sum_{s=1}^{\infty} \sigma^s \Lambda_{t,t+s} \lambda^{s-1} \left(\prod_{j=1}^{s} (1 - \varphi_{i,t+j}) \right) A_{t+s} \left(\Delta^h_{t+s} - \Delta^l_{t+s} \right) K^{\alpha-1}_{t+s} \right) \\
&= P_t \sum_{i \in \mathcal{L}} x_t,
\end{aligned}
$$

where σ^s is the probability that the has not suffer exogenous exit shock between period t and $t+s$, $\prod_{j=1}^{s} (1 - \varphi_{i,t+j})$ is the probability that between period t and $t+s$, the firm i has not defaulted on the provided implicit recourse. Due to the mentioned property of the model, with the exception of exogenous exit, firms' defaults on implicit recourse are synchronized an take place in states with high dispersion in assets qualities (i.e., in deep recessions). This allows us to rewrite the probability term of the above expression as $\prod_{j=1}^{s} (1 - \chi D, t + j)$, where $\chi_{D,t+1} = 1$ when the state in which all firms due to sufficiently negative large aggregate shock default. P_t is the present value of costs of providing implicit recourse per unit of investment, which can be written recursively

$$
\begin{aligned}
P_t &= \sum_{s=1}^{\infty} \sigma^s \Lambda_{t,t+s} \lambda^{s-1} \left(\prod_{j=1}^{s} (1 - \chi_{D,t+1}) \right) A_{t+s} \left(\Delta^h_{t+s} - \Delta^l_{t+s} \right) K^{\alpha-1}_{t+s} \\
&= \sigma \Lambda_{t,t+1} (1 - \chi_{D,t+1}) \left[A_{t+1} \left(\Delta^h_{t+1} - \Delta^l_{t+1} \right) K^{\alpha-1}_{t+1} + \lambda P_{t+1} \right].
\end{aligned}
$$

On the aggregate, there is an outstanding stock of implicit recourse obligations

$$
SIR_t = \sum_i sir_{i,t} = \left(1 - f^{NIR}_t \right) (1 - \omega_t) K_t P_t,
$$

where ω_t is the share of low quality securitized assets in the total stock of capital K_t and f^{NIR}_t is the share of low quality securitized assets which does not bear implicit recourse either because they were not provided or they were defaulted upon in the past.

---

[34]Note that I sum all the new investment carried out in this period by all the issuers with access to low quality projects and not only the sold part of their investment. This is because the "skin in the game" constraint holds only for one period. In the following periods, the remaining part of the investment can be sold, but it still has to carry the implicit guarantee.

A.2 Arbitrage

In order to solve the model and maintain its tractability, I introduce a possibility of arbitrage in the model, which is in the spirit similar to the one in Gertler and Kiyotaki (2010). The assumption allows me not to keep track of the distribution of firms' stock of implicit recourse obligations as well as of firms' equity.

Uniform distribution of equity as well as implicit support obligations across islands maximizes the ex ante return given the i.i.d. nature of the investment shock. Firms with a higher than average stock of implicit obligations would be at a disadvantage compared with others. Therefore, it is optimal for them to equalize the ratio of stock of implicit obligations to equity too.

The process works as follows. A fraction of firms from islands with high level of equity move to islands with a low level of equity. On entry to the island, they can privately observe the stock of implicit obligations still kept on the island[35]. If the ratio of this stock to equity is higher than the average in the economy, they will decide not to enter. Such islands would remain with a low level of equity compared to others, which would reveal to everyone that there is a high stock of implicit obligations on the island, and would hinder the ability of the firm located on the island to sell securitized assets and exploit potential investment opportunities. Anticipating such a development, firms find it optimal to pay for the transfer of some of their stock of implicit obligations to other firms, or accept payment for receiving some additional stock of implicit obligations prior to the redistribution of equity.

A.3 Derivation of firms value functions and market clearing conditions

To obtain the respective market clearing condition, let's first rewrite the firm's value functions recursively:

$$
\begin{aligned}
V_{i,t}^{ND}(n_t; S_t) &= \max E_t\{(1-\sigma)n_{i,t} + \sigma\Lambda_{t,t+1}[\varphi_{i,t+1}V_{i,t+1}^{ND}(n_{i,t+1}^{ND}; S_{t+1}) \\
&\quad + (1-\varphi_{i,t+1})p_{i,t}V_{i,t+1}^{D}(n_{i,t+1}^{D}; S_{t+1}) \\
&\quad + (1-\varphi_{i,t+1})(1-p_{i,t})V_{i,t+1}^{ND}(n_{i,t+1}^{D}; S_{t+1})]\}, \quad (15) \\
V_{i,t}^{D}(n_{i,t}; S_t) &= \max E_t\{(1-\sigma)n_{i,t} + \sigma\Lambda_{t,t+1}V_{i,t+1}^{D}(n_{i,t+1}; S_{t+1})\}, \quad (16)
\end{aligned}
$$

for the firm with a reputation of not-defaulting on implicit recourse and for the firm which has defaulted already in the past and suffers the trigger punishment. Note that when the firm is punished for defaulting, $p_{i,t} = 1$, and when the firm is not punished after defaulting, $p_{i,t} = 0$.[36] I guess and verify that $V_{i,t}^{ND}(n_{i,t}) = n_{i,t}\nu_t^{ND}$ and

---

[35]Note that the stock of implicit obligations cannot be observed publicly, otherwise, the distribution of investment opportunities would also become public information.

[36] Following the discussion in section 2.3, where I suggest that firms in equilibrium do not individually default on implicit recourse but there are states where it is optimal for all firms to default on the implicit recourse (sin these states $\chi_{i,t+1}^{D} \equiv 1$, while in all other states $\chi_{i,t+1}^{D} \equiv 0$), this implies that $\varphi_{i,t+1} = \chi_{t+1}^{D}\forall i$, $(1-\varphi_{i,t+1})p_{i,t} = 0\forall i$ and $(1-\varphi_{i,t+1})(1-p_{i,t}) = 1 - \chi_{t+1}^{D}\forall i$

$V_{i,t}^D (n_{i,t}) = n_{i,t}\nu_t^D$. From this guess, obtain

$$\nu_t^{ND} = E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \left[\chi_{t+1}^D \frac{n_{i,t+1}^{ND}}{n_{i,t}} \nu_{t+1}^{ND} + \left(1 - \chi_{t+1}^D \right) \frac{n_{t+1}^{DD}}{n_t} \nu_{t+1}^D \right] \right\}, \quad (17)$$

$$= E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \left[\chi_{t+1}^D R_{t+1}^{n,ND} \nu_{t+1}^{ND} + \left(1 - \chi_{t+1}^D \right) R_{t+1}^{n,DD} \nu_{t+1}^D \right] \right\}, \quad (18)$$

for the value of equity of a firm with a reputation of not-defaulting on implicit recourse and

$$\nu_t^D = E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} \frac{n_{t+1}}{n_t} \nu_{t+1}^D \right\},$$

$$\nu_t^D = E_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} R_{t+1}^{n,D} \nu_{t+1}^D \right\}, \quad (19)$$

where $R_{t+1}^{n,ND}$, $R_{t+1}^{n,DD}$, $R_{t+1}^{n,D}$ are the next period return on equity for a firm which does not default on implicit recourse, a firm which has defaulted and suffers a punishment, and a firm which defaulted in the past and suffers the punishment, respectively.

To derive the capital goods market clearing condition, we maximize the above value function conditional on observed realization of the i.i.d. investment shock. In this case, the return on equity of an individual firm may differ depending on the investment opportunity. However, due to arbitrage, the next period marginal value of equity will be equal across firms $\nu_{i,t+1}^{ND} = \bar{\nu}_{t+1}^{ND}$ and $\nu_{i,t+1}^D = \bar{\nu}_{t+1}^D$, where $\bar{\nu}_{t+1}^{ND}$ and $\bar{\nu}_{t+1}^D$ denote the value of equity for the aggregate sector of financial firms of the respective type.

Note that, due to logarithmic utility function, we can show that $E_t \left(\bar{\nu}_{t+1}^{ND} \right)$ is a constant. To demonstrate this, we can compute $\bar{\nu}_t^{ND}$ from (18) but taking the expectations before the arrival of the i.i.d. investment shock when the expected return on equity R_{t+1}^{ND} is equal across firms:

$$\bar{\nu}_t^{ND} = \bar{E}_t \left\{ (1 - \sigma) + \sigma \Lambda_{t,t+1} R_{t+1}^{ND} \bar{\nu}_{t+1}^{ND} \right\},$$

$$= \bar{E}_t \left\{ (1 - \sigma) + \frac{\beta \sigma}{\sigma + \xi} \bar{\nu}_{t+1}^{ND} \right\},$$

$$= \frac{1 - \sigma}{1 - \frac{\beta \sigma}{(\sigma + \xi)}}.$$

Maximizing such a transformed value function with respect to the choice of various capital goods, we obtain standard Euler equations (6).

A.4 Trigger punishment strategy

Credibility of the punishment. A necessary condition for the existence of the equilibrium in which credible and therefore valuable implicit recourse is being provided is the credibility of the punishment rule. Any firm, which observes default on the implicit recourse by another firm, has to prefer punishing the defaulting firm rather

than non-punishing it, even ex post. This is expressed in condition (5).

Below I derive analytically both elements of that inequality in the case of the pooling deterministic steady state without storage. In the fully stochastic version, this can be solved numerically. Following the same steps as in Appendix A.3, we can find that the value function of the firm that always punished, and therefore has a reputation of being a "tough investor", is:

$$V_i^P(n) = \bar{\nu}^P n_i = (1-\sigma)\left(1 - \frac{\beta\sigma}{\sigma+\xi}\left(\pi\mu R'^{h,IR} + \pi(1-\mu)R'^{l,IR} + (1-\pi)R'^s\right)\right)^{-1} n_i,$$

and the value function of the firm that failed to punish and therefore lost its reputation of being a "tough investor" is:

$$V_i^{NP}(n) = \bar{\nu}^{NP} n_i = (1-\sigma)\left(1 - \frac{\beta\sigma}{\sigma+\xi}\left(\pi\mu R'^{h,IR} + \pi(1-\mu)R'^{l,IR} + (1-\pi)R'^{s,NP}\right)\right)^{-1} n_i,$$

The difference in the above equations is the return obtained in the case that firms do not have direct access to new productive projects and have to rely on the financial intermediation (R'^s vs. $R'^{s,NP}$). I consider an equilibrium, where if a firm fails to punish while other firms do punish a default on implicit recourse, then such a firm loses its "reputation of being a tough investor". As a result, other firms will expect that this firm will never punish in the future. As a consequence, they will never again honor implicit support provided on the primary market to such firm. Note that in a pooling equilibrium, it is not possible for such a firm to make sure it buys assets without implicit recourse. Issuers of low quality assets sell on primary markets and by providing implicit recourse, they try to mimic high quality assets. They will not agree to sell low quality assets without implicit recourse for a lower price as this would reveal their type. Therefore, when a firm without the "reputation of being a tough investor" buys assets on the primary market in a pooling equilibrium, its return is $R'^{s,NP} = \frac{\mu x'^h + (1-\mu)x'^l}{q^p}$. While firms with a "tough investors reputation" have a return of $R'^{s,NP} = \frac{\mu x'^h + (1-\mu)x'^{l,G}}{q^p}$.[37]

The firm without a "tough investor reputation" may try to buy assets on the secondary (resale) markets, but even here they may be in a disadvantageous position. As their outside option is primary market or storage, I assume that selling firm may discriminate and charge them a higher than price q^s. The price for which an asset is sold on the secondary market to a firm without reputation is then somewhere on the interval $q^{s,NP} \in [q^s, q^{s,max}]$, depending on the bargaining power of sellers and buyers without "tough investor reputation". The maximum price that can be charged on the secondary market is given by their outside option, i.e. primary market or storage $q^{s,max} = x'^s / \max\left(R^{s,NP}, R^z\right)$, where x^s are the next period cash flows an value of the assets bought on the secondary market.[38] Therefore, unless all bargaining power is on

---

[37] x'^h, x'^l and $x'^{l,G}$ are the next period cash flows and values for assets of high quality, of low quality without implicit recourse and of low quality with implicit recourse, respectively. Precise definition is in Appendix B.1.

[38] Note that unlike in Kuncl (2015), the storage option limits the degree to which conditions on the secondary market to firms without "tough investor reputation" are worse compared to firms with

the side of firms without reputation, then $\bar{\nu}^P > \bar{\nu}^{NP}$, and therefore, saving firms have incentives to punish, and inequality (5) would be satisfied at least in some states of the world.

Limits of the punishment   In some states of the world the inequality (5) will not be satisfied. In those states all firms default on all the existing implicit recourse and the punishment is not triggered. The reason is that in those states of the world the trigger punishment strategy is not renegotiation-proof.

First lets consider a situation in which trigger punishment strategy is renegotiation-proof. Suppose that one single firm has just defaulted on the outstanding implicit recourse. When firms decide whether to punish it, they can agree with it on preferential trade conditions. Instead of punishing the defaulting firm, they can negotiate better terms with the defaulted firm, i.e., buy the assets from them for a lower than market price $q^{p,RN} < q^p$, obtaining thus a higher return $R^{s,RN} > R^s$. However, those benefits from renegotiation are limited by the fact that the defaulted firm would be selling the assets only with probability π, and the quantity of assets that the firm can sell is limited proportionally to its equity. Even if the quantity of the assets sold by the defaulted firm is large enough, renegotiation would not be optimal as long as

$$R^s > \pi R^{s,RN} + (1 - \pi) R^{s,NP}.$$

This depends on prices $q^h, q^{h,NP}, q^{h,RN}$, which themselves depend upon the relative bargaining power of different agents in the economy.

Now lets consider a case, when more firms default. This will in particular happen when all firms, which have access to low quality investment opportunities in a boom period, decide to invest, securitize and sell the newly issued assets but the following period the economy moves to a deep recession. All those firms find it unilaterally optimal to default on the large outstanding implicit guarantees even when expecting to suffer a punishment. When a large fraction of firms defaults, negotiating with them better terms of trade is more attractive since by law of large numbers in each period some of these firms will have access to new high quality investment opportunities. Therefore, in this case punishment strategy is not renegotiation proof.

Note that when some firms stop to punish, other firms that have not yet defaulted on the implicit recourse will stop expecting a punishment and will also default on the implicit recourse.

A.5   Role of the "skin in the game" constraint

When the **"skin in the game" constraint is not binding**, then, due to competition, prices of high quality assets are equal to the unitary costs of financing high quality

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such reputation. But in some states of the world, in particular in the boom, the storage is not used as it has a lower return than buying securitized assets. In these states firms without "tough investor reputation" may face worse conditions on the secondary market

projects $q_t^h = 1$. Firms do not make profits from securitizing part of their investment. Therefore, firms with access to low quality projects do not have incentives to mimic firms with high quality investment opportunities. In the steady state, the consumption goods market clearing condition $Y_t = X_t + C_t$ becomes

$$r^h = (1 - \lambda) + (1 - \sigma - \xi)\left(r^h + \lambda\right),$$
$$r^h + \lambda = \frac{1}{\sigma + \xi} > \frac{1}{\beta}.$$

Due to the binding exit shock, where by assumption $\sigma + \xi < \beta$[39], the return to investment is higher than in the first-best, and therefore, there is underinvestment. In this case, only high quality loans are being financed $\omega = 1$ and storage technology is not used $Z = 0$.

When the **"skin in the game" constraint becomes binding**, the supply of securitized cash flows from projects becomes limited, which drives their market price above the unitary costs of refinancing. The outcome with only this friction binding is analogous to Kuncl (2015), where you can find a precise definition of steady state under different levels of the parameter θ. Here, I derive only the condition, which makes the "skin in the game" constraint binding, so that the prices of high quality projects exceeds the unitary costs but low quality projects are still not financed in equilibrium.

The level of aggregate investment becomes determined by the constraint:

$$X_t = \frac{\pi \mu (\sigma + \xi) K_t \left(r_t^h + \lambda q_t^h\right)}{\left(1 - \theta q_t^h\right)},$$

which in the steady state becomes $(1 - \lambda)\left(1 - \theta q^h\right) = \pi \mu (\sigma + \xi)\left(r^h + \lambda q^h\right)$. The consumption goods market clearing condition in the steady state takes the form

$$r^h = (1 - \lambda) + (1 - \sigma - \xi)\left(r^h + \lambda q^h\right). \tag{20}$$

Combining these equations, we can obtain the expression for the steady state price of high quality assets

$$q^h = \frac{(1 - \lambda)(1 - \pi \mu)}{(1 - \lambda)\theta + \pi \mu \lambda}. \tag{21}$$

Should the price exceed one, we can derive from (21) that we need a large enough skin in the game

$$1 - \theta > \frac{\pi \mu}{(1 - \lambda)}. \tag{22}$$

A binding "skin in the game" is a precondition for the **use of storage technology**. When (22) is not binding, then $q^h = 1$ and the profits from investment are equally

---

[39]Note that if $\sigma + \xi \geq \beta$, then the exit shock would not be binding since households would decide to distribute more dividends than those obtained by the exit shock.

shared by all firms. Binding (22) increases the returns for investing and securitizing firms and lowers the return for saving firms. But even when $q^h > 1$, storage would not be used if the "skin in the game" constraint does not exceed the level needed to bring the return from buying securitized loans to the unit return from storage:

$$R^h > R^z,$$
$$\frac{r^h + \lambda q^h}{q^h} > 1.$$

This condition can be rewritten using (20) and (21) to

$$\frac{r^h + \lambda q^h}{q^h} = \frac{(1-\lambda)}{(\sigma+\xi)\, q^h} + \frac{\lambda}{(\sigma+\xi)} > 1,$$
$$\frac{(1-\lambda)\,\theta + \lambda}{(1-\pi\mu)\,(\sigma+\xi)} > 1,$$
$$(1-\lambda)(1-\theta) < (\sigma+\xi)\,\pi\mu + 1 - \sigma - \xi. \tag{23}$$

Since $\sigma + \xi < 1$, then $\pi\mu < 1 - (1-\pi\mu)(\sigma+\xi)$, and therefore, there is a non-empty interval of parameter θ such that both (22) and (23) are satisfied. In other words, the "skin in the game" constraint consistent with positive amount of storage, i.e., not satisfying the condition (23), has to be stricter than condition (22).

If the condition (23) is not satisfied, then $Z > 0$ and the market clearing conditions become

$$r^h + z = (1-\lambda) + (1-\sigma)\,\lambda\left(r^h + \lambda q^h + z\right) + z,$$
$$\frac{r^h + \lambda q^h}{q^h} = 1,$$

for the consumption goods market and the capital goods market, respectively. Note that $z \equiv Z/K$ is the ratio of the level of storage to capital. The investment function becomes

$$(1-\lambda)\left(1 - \theta q^h\right) = \pi\mu\,(\sigma+\xi)\left(r^h + \lambda q^h + z\right).$$

Combining the two market clearing conditions, we obtain

$$(\sigma + \xi - \lambda)\,q^h = 1 - \lambda + (1 - \sigma - \xi)\,z,$$

and combining the investment function with the capital goods market clearing condition, we obtain

$$(1-\pi\mu)(1-\lambda) = (\theta(1-\lambda) + \lambda\pi\mu)\,q^h + \pi\mu z.$$

From this system of 2 equations with 2 unknowns, we obtain

$$q^h = \frac{(\sigma+\xi)\,\pi\mu + 1 - \sigma - \xi}{(\sigma+\xi)\,\pi\mu + (1-\sigma-\xi)\,\theta},$$

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and

$$z = \frac{(\sigma + \xi)(1 - \pi\mu) - \lambda - \theta - \lambda\theta}{(\sigma + \xi)\pi\mu + (1 - \sigma - \xi)\theta}.$$

Note that for lower θ, π, μ or λ the economy is more constrained, and therefore both q^h and z would increase in the steady state.

A.6 Separating condition without provision of implicit recourse

When we introduce asymmetric information in the primary market for securitized products, there may still be a separating equilibrium, in which firms with access to low quality projects find it optimal to buy high quality projects rather than investing and securitizing cask flows from the low quality projects and selling these for the best possible price, i.e., for the market price for high quality projects. The condition for the existence of such a separating equilibrium is in the steady state:

$$
\begin{aligned}
V\,(buying\,high\,projects) &\geq V\,(mimicking)\,, \\
R\,|\,buying\,high\,projects &\geq R\,|\,mimicking. \quad (24)
\end{aligned}
$$

When the "skin in the game" is not binding, then this condition $(r^h + \lambda \geq r^l + \lambda q^l)$ is always satisfied. Note that using the market clearing condition for capital goods markets $A^h/q^h = A^l/q^l$, this condition can be rewritten to

$$\frac{A^h}{A^l} > 1.$$

When the "skin in the game" is binding, condition (24) becomes

$$
\begin{aligned}
\frac{r^h + \lambda q^h}{q^h} &\geq \frac{r^l + \lambda q^l}{\frac{1 - \theta q^h}{1 - \theta}}, \\
\frac{r^h + \lambda q^h}{r^l + \lambda q^l} = \frac{q^h}{q^l} &\geq \frac{(1 - \theta)\,q^h}{1 - \theta q^h}.
\end{aligned}
$$

If the condition (23) is satisfied, i.e., storage is not used in equilibrium, substituting for q^h from (21), the separating condition becomes

$$\frac{A^h}{A^l} \geq \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu}. \quad (25)$$

When storage is not used in equilibrium, a higher share of high quality projects $\pi\mu$, a lower skin in the game $(1 - \theta)$ or smaller depreciation rate $(1 - \lambda)$ would decrease the RHS of (25), and therefore, it will be easier to satisfy the separating condition.

If storage is used in equilibrium substituting for q^h from (21), the separating condi-

tion becomes

$$\frac{A^h}{A^l} \geq \frac{(\sigma + \xi)\,\pi\mu + 1 - \sigma - \xi}{(\sigma + \xi)\,\pi\mu}. \tag{26}$$

In this case, the higher share of high quality projects $\pi\mu$, the higher rate of survival of financial firms σ or higher equity share of new firms ξ, the lower the RHS of (26) is and the more likely it would be to satisfy the separating condition.

A.7   Separating condition with provision of informative implicit recourse

Separating equilibrium condition when the provided implicit recourse is informative is

$$V\,(buying\,high\,projects) \geq V\,(mimicking\,without\,defaulting). \tag{27}$$

When implicit recourse is being provided, there are two outcomes possible. Either the condition (4) is satisfied, then

$$V\,(mimicking\,without\,defaulting) \geq V\,(mimicking\,and\,defaulting),$$

and the signal in the form of implicit recourse is informative, or (4) is not satisfied, then

$$V\,(mimicking\,without\,defaulting) < V\,(mimicking\,and\,defaulting),$$

and the signal is not informative. We concentrate on the prior case of informative signal otherwise the existence of separating equilibrium condition collapses to (6).

Separating equilibrium conditions when the provided implicit recourse is informative is in the steady state

$$\begin{aligned}
V\,(buying\,high\,projects) &\geq V\,(mimicking\,without\,defaulting), \\
R \mid buyin\,high\,projects &\geq R \mid mimicking\,without\,defaulting, \\
\frac{r^h + \lambda q^h}{q^h} &\geq \frac{r^l + \lambda q^l - \frac{1}{1-\theta}P}{\frac{1-\theta q^h}{1-\theta}}.
\end{aligned}$$

After substituting for the steady state cost of keeping the steady state promise $P = \beta\sigma\left(r^h - r^l + \lambda P\right) = \beta\sigma\left(r^h - r^l\right)/\left(1 - \beta\sigma\lambda\right)$, we obtain

$$\frac{1 - \theta q^h}{(1-\theta)\,q^h} \geq \frac{r^l + \lambda q^l - \frac{\beta\sigma\left(r^h - r^l\right)}{(1-\theta)(1-\beta\sigma\lambda)}}{r^h + \lambda q^h},$$

$$\frac{1 - \theta q^h}{(1-\theta)\,q^h} \geq \frac{q^l}{q^h} - \frac{\beta\sigma}{(1-\theta)\,(1-\beta\sigma\lambda)}\,\frac{\frac{r^h}{q^h}\left(1 - \frac{q^l}{q^h}\right)}{\frac{r^h + \lambda q^h}{q^h}}. \tag{28}$$

To simplify the above expression, we need to find the expression for r^h/q^h. In the

case when the "skin in the game" constraint is not sufficiently binding to have positive storage in equilibrium, then by combining the relevant steady state market clearing condition and the investment function, we obtain

$$\begin{aligned}
\frac{r^h}{q^h} &= \frac{1-\lambda}{q^h(\sigma+\xi)} + \frac{(1-\sigma-\xi)}{(\sigma+\xi)}\lambda, \\
&= \frac{\theta(1-\lambda) + \lambda(1-\sigma-\xi) + \pi\mu\lambda(\sigma+\xi)}{(1-\pi\mu)(\sigma+\xi)}.
\end{aligned}$$

Substituting this and the expression for q^h from (21) into (28), we obtain

$$\frac{\pi\mu\lambda + (1-\lambda)\theta\pi\mu}{(1-\pi\mu)(1-\lambda)(1-\theta)} \geq \frac{q^l}{q^h} - B\left(1 - \frac{q^l}{q^h}\right), \tag{29}$$

where

$$B = \frac{\beta\sigma(\theta + \lambda(1-\theta-(1-\pi\mu)(\sigma+\xi)))}{(1-\theta)(1-\beta\sigma\lambda)(\theta+\lambda(1-\theta))}.$$

The inequality (29) after substitution of $A^h/q^h = A^l/q^l$, becomes

$$\begin{aligned}
\frac{A^h}{A^l} &\geq \frac{1+B}{\frac{\pi\mu\lambda+(1-\lambda)\theta\pi\mu}{(1-\pi\mu)(1-\lambda)(1-\theta)} + B} \\
\frac{A^h}{A^l} &\geq \frac{(1-\pi\mu)(1-\lambda)(1-\theta)(1+B)}{\pi\mu\lambda + (1-\lambda)\theta\pi\mu + B(1-\pi\mu)(1-\lambda)(1-\theta)}. \tag{30}
\end{aligned}$$

In the case when there is a positive level of storage in equilibrium, we can depart from the capital asset market clearing condition $\left(r^h + \lambda q^h\right)/q^h = 1$ to transform (28) into

$$\frac{(\sigma+\xi)\pi\mu}{(\sigma+\xi)\pi\mu + 1-\sigma-\xi} \geq \frac{q^l}{q^h} - B\left(1 - \frac{q^l}{q^h}\right),$$

where

$$B = \frac{\beta\sigma(1-\lambda)}{(1-\theta)(1-\beta\sigma\lambda)},$$

which becomes

$$\frac{A^h}{A^l} \geq \frac{((\sigma+\xi)\pi\mu + 1-\sigma-\xi)(1+B)}{(\sigma+\xi)\pi\mu + B((\sigma+\xi)\pi\mu + 1-\sigma-\xi)}. \tag{31}$$

When $B = 0$, conditions (30) and (31) collapse to (6) and (7), respectively. To prove that (30) is less strict then (6), we have to prove that the RHS of (30) is increasing

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with B:

$$\frac{\partial}{\partial B} \frac{(1 - \pi\mu)(1 - \lambda)(1 - \theta)(1 + B)}{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu + B(1 - \pi\mu)(1 - \lambda)(1 - \theta)}$$
$$= \frac{\pi\mu\lambda + (1 - \lambda)\theta\pi\mu - (1 - \pi\mu)(1 - \lambda)(1 - \theta)}{\frac{(\pi\mu\lambda + (1 - \lambda)\theta\pi\mu + B(1 - \pi\mu)(1 - \lambda)(1 - \theta))^2}{(1 - \pi\mu)(1 - \lambda)(1 - \theta)}} \quad < \quad 0.$$

The last inequality comes from the fact that $q^h > 1$, and therefore, the RHS of (6) also exceeds 1. Similarly, we can show that the RHS of (31) is increasing with B:

$$\frac{\partial}{\partial B} \frac{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)(1 + B)}{(\sigma + \xi)\pi\mu + B((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}$$
$$= \frac{(\sigma + \xi)\pi\mu - ((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}{\frac{((\sigma + \xi)\pi\mu + ((\sigma + \xi)\pi\mu + 1 - \sigma - \xi))^2}{((\sigma + \xi)\pi\mu + 1 - \sigma - \xi)}} \quad < \quad 0.$$

A.8 Adverse selection in resale markets

Case without implicit recourse: Prices depend on the share of high quality assets We derive the pricing conditions from the F.O.C. of saving firms (subset $\mathcal{S}_t$) in a pooling equilibrium. The value of a high quality asset q_t^h reflects the expected gross profit next period and the value of the asset next period, which is q_{t+1}^h if the firm has no investment opportunities and keeps the asset on the balance sheet, or q_{t+1}^s if the firms has an investment opportunity and sells the asset. The Euler condition below shows the marginal indifference of the saving firm between keeping a high quality asset or buying an asset on the primary market:

$$E_t\left[\Lambda_{t,t+1} \frac{r_{t+1}^h + \lambda\pi q_{t+1}^s + \lambda(1 - \pi) q_{t+1}^h}{q_t^h}\right] = E_t\left[\Lambda_{t,t+1} R_{t+1}^p\right],$$

where the expected return of an asset bought on the primary market is

$$E_t\left(\frac{\mu\left(r_{t+1}^h + \lambda\left(\pi q_{t+1}^s + (1 - \pi) q_{t+1}^h\right)\right) + (1 - \mu)\left(r_{t+1}^l + \lambda q_{t+1}^s\right)}{q_t^p}\right).$$

An asset bought on the primary market in the pooling equilibrium is, with probability μ, of high quality and with probability $1 - \mu$ of low quality.

The value of the low quality asset reflects the expected next period gross profits and the expected next period resale price since low assets are always sold on the resale market

$$E_t\left[\Lambda_{t,t+1} \frac{r_{t+1}^l + \lambda q_{t+1}^s}{q_t^l}\right] = E_t\left[\Lambda_{t,t+1} R_{t+1}^p\right].$$

The price of an asset sold on the resale market satisfies:

$$E_t\left[\Lambda_{t,t+1}\frac{f_t^h\left(r_{t+1}^h+\lambda\left(\pi q_{t+1}^s+(1-\pi)\,q_{t+1}^h\right)\right)+\left(1-f_t^h\right)\left(r_{t+1}^l+\lambda q_{t+1}^s\right)}{q_t^s}\right]=E_t\left[\Lambda_{t,t+1}R_{t+1}^p\right],$$

where f_t^h is the share of high quality assets sold on the resale market in this period, which is in the case of a pooling equilibrium

$$f_t^h=\frac{\pi\omega_t}{\pi+(1-\pi)\,(1-\omega_t)}.$$

Case without implicit recourse: Conditions for no trade in high quality assets Investing firms prefer to keep their high quality loans rather than to sell them and invest the obtained liquidity if the following condition is satisfied in the steady state:

$$\begin{aligned}
V\,(keeping\,high\,projects) &\geq& V\,(selling\,high\,projects)\,,\\
R\,(keeping\,high\,projects) &\geq& R\,(selling\,high\,projects)\,,\\
r^h+\lambda\pi\mu q^s+\lambda\,(1-\pi\mu)\,q^h &\geq& q^s\frac{r^h+\lambda\pi\mu q^s+\lambda\,(1-\pi\mu)\,q^h}{\frac{1-\theta q^p}{1-\theta}}\,,\\
\frac{1-\theta q^p}{1-\theta} &\geq& q^s=f^h q^h+\left(1-f^h\right)q^l\,,\\
1-\theta\left(\mu q^h+(1-\mu)\,q^l\right) &\geq& (1-\theta)\left(f^h q^h+\left(1-f^h\right)q^l\right)\,,\\
1-\theta\mu q^h-(1-\theta\mu)\,q^l &\geq& f^h\,(1-\theta)\left(q^h-q^l\right)\,,\\
f^h &\leq& \frac{1-\theta\mu q^h-(1-\theta\mu)\,q^l}{(1-\theta)\,(q^h-q^l)}.
\end{aligned}$$

Case with implicit recourse: Prices depend on the share of assets without implicit recourse We derive the pricing conditions from the F.O.C. of saving firms in a pooling equilibrium. In contrast to the case without implicit recourse, the prices depend on the share of low quality assets without implicit recourse f^{NIR}. The shadow value of a high quality asset remains the same

$$E_t\left[\Lambda_{t,t+1}\frac{r_{t+1}^h+\lambda\pi q_{t+1}^s+\lambda\,(1-\pi)\,q_{t+1}^h}{q_t^h}\right]=E_t\left[\Lambda_{t,t+1}R_{t+1}^p\right],$$

where the expected return of an asset bought on the primary market is $R_{t+1}^p=E_t\left(\frac{x_{t+1}^p}{q_t^p}\right)$, where

$$\begin{aligned}
x_{t+1}^p &=& \mu\left(r_{t+1}^h+\lambda\left(\pi q_{t+1}^s+(1-\pi)\,q_{t+1}^h\right)\right)+(1-\mu)\left(\chi_{D,t+1}r_{t+1}^l+(1-\chi_{D,t+1})\,r_{t+1}^h\right)\\
&& +(1-\mu)\,\lambda\left((1-\pi)\,(1-\chi_{D,t+1})\,q_{t+1}^l+(1-(1-\pi)\,(1-\chi_{D,t+1}))\,q_{t+1}^s\right).
\end{aligned}$$

An asset bought on the primary market in the pooling equilibrium is with probability μ of high quality and with probability $1 - \mu$ of low quality. The implicit recourse on the low quality asset may be provided and then the asset generates cash flow r_{t+1}^h, or recourse may be defaulted upon and then the asset generates cash flow r_{t+1}^l. In a pooling equilibrium, assets will be sold on the resale market in order to take advantage of the investment opportunity with probability π. If the implicit recourse is defaulted upon $(\chi_{D,t+1} = 1)$, holders will be able to identify the low quality assets and will sell them on the resale markets. Otherwise assets are kept on the balance sheet (high quality assets with probability $1 - \pi$ and low quality assets with probability $(1 - \pi)(1 - \chi_{D,t+1})$) and valued by their shadow price q^h or q^l.

As already mentioned, the low quality assets are either non-identified or without implicit recourse. The value of the low quality asset without implicit recourse is q_t^s since it is never kept on the balance sheet till the next period and it is immediately sold in the resale market. The shadow value of the low quality assets which are a non-identified part of the firm's portfolio with implicit recourse is determined by:

$$E_t \left[\Lambda_{t,t+1} \frac{x_{t+1}^l}{q_t^l} \right] = E_t \left[\Lambda_{t,t+1} R_{t+1}^p \right],$$

where

$$\begin{aligned}
x_{t+1}^l &= \chi_{D,t+1} r_{t+1}^l + (1 - \chi_{D,t+1}) r_{t+1}^h \\
&\quad + \lambda (1 - \pi)(1 - \chi_{D,t+1}) q_{t+1}^l + \lambda (1 - (1 - \pi)(1 - \chi_{D,t+1})) q_{t+1}^s.
\end{aligned}$$

The price of an asset sold on the resale market satisfies:

$$E_t \left[\Lambda_{t,t+1} \frac{x_{t+1}^s}{q_t^s} \right] = E_t \left[\Lambda_{t,t+1} R_{t+1}^p \right],$$

where

$$\begin{aligned}
x_{t+1}^s &= f_t^h \left(r_{t+1}^h + \lambda \left(\pi q_{t+1}^s + (1 - \pi) q_{t+1}^h \right) \right) \\
&\quad + \left(1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1}) \right) r_{t+1}^l + f_t^{l,IR} (1 - \chi_{D,t+1}) r_{t+1}^h \\
&\quad + \lambda (1 - \pi) f_t^{l,IR} (1 - \chi_{D,t+1}) q_{t+1}^l + \lambda \left[(1 - \pi) \left(1 - f_t^h - f_t^{l,IR} (1 - \chi_{D,t+1}) \right) + \pi \right] q_{t+1}^s,
\end{aligned}$$

where f_t^h is the share of high quality assets sold on the resale market in this period, which is in the case of a pooling equilibrium:

$$f_t^h = \frac{\pi \omega_t}{\pi + f_t^{NIR} (1 - \pi)(1 - \omega_t)},$$

and $f_t^{l,IR}$ is the share of low quality assets on the resale markets which bears implicit

recourse

$$f_t^{l,IR} = \frac{\pi (1 - \omega_t) \left(1 - f_t^{NIR}\right)}{\pi + f_t^{NIR} (1 - \pi) (1 - \omega_t)}.$$

In the steady state with informative implicit recourse, firms default only when they exit, i.e., with probability σ. The share of low quality assets without implicit recourse (out of all low quality assets) after the decisions on the default on implicit recourse f_t^{NIR} is then given by

$$f_t^{NIR} = f_t^D,$$

where f_{t+1}^D is the share of low quality assets with defaulted implicit recourse at the end of the period t is

$$f_{t+1}^D = \left(f_t^{NIR} + 1 - \sigma\right) \frac{\lambda K_t}{\lambda K_t + X_t}.$$

This gives us the steady state level $f^{NIR} = \lambda (1 - \sigma) / (1 - \lambda)$. In the next section, we will show that when economy moves to a deep recession, there will be systemic default on implicit recourse given and $f_t^{NIR} = 1$.

B Markov-switching regimes

B.1 Equilibrium conditions

The investment function:

$$X_t = \frac{\chi_{1,t} (\sigma + \xi) \left[\left(\omega_t r_t^h + (1 - \omega_t) r_t^l + \lambda q_t^s\right) K_t + Z_t\right]}{1 - \theta q_t^p}.$$

Consumption function:

$$\begin{aligned}
C_t =\ & (1 - \sigma - \xi) \left[\left(\omega_t r_t^h + (1 - \omega_t) r_t^l\right) K_t \right. \\
& \left. + \lambda K_t \left(\omega_t q_t^h + \left(1 - f_t^{NIR}\right) (1 - \omega_t) q_t^l + f_t^{NIR} (1 - \omega_t) q_t^s\right) + Z_t\right].
\end{aligned}$$

The consumer goods market clearing condition:

$$\left(\omega_t r_t^h + (1 - \omega_t) r_t^l\right) K_t + Z_t = X_t + C_t + Z_{t+1}.$$

Law of motion for capital

$$K_{t+1} = \lambda K_t + X_t,$$

and law of motion for the share of high quality assets

$$\omega_{t+1} = \frac{H_{t+1}}{K_{t+1}} = \frac{\lambda H_t + X_t^H}{\lambda K_t + X_t} = \frac{\omega_t \lambda K_t + \chi_{2,t} X_t}{\lambda K_t + X_t} = \frac{\omega_t \lambda + \chi_{2,t} X_t / K_t}{\lambda + X_t / K_t}.$$

Capital goods market clearing conditions: I calibrate the model such that there is a positive amount of storage as well as investment in the economy. Since the return on

storage is $R_{t+1}^Z = 1$, the market clearing conditions are the following:

$$E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^h}{q_t^h}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^l}{q_t^l}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^p}{q_t^p}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^s}{q_t^s}\right] = E_t\left[\Lambda_{t,t+1}\right],$$

where the next period cash flows and values of these assets are defined

$$
\begin{aligned}
x_{t+1}^h &= r_{t+1}^h + \lambda\chi_{1,t+1}q_{t+1}^s + \lambda\left(1 - \chi_{1,t+1}\right)q_{t+1}^h, \\
x_{t+1}^l &= \pi_{D,t+1}r_{t+1}^l + \left(1 - \pi_{D,t+1}\right)r_{t+1}^h \\
&\quad + \lambda\left(1 - \chi_{1,t+1}\right)\left(1 - \pi_{D,t+1}\right)q_{t+1}^l + \lambda\left(1 - \left(1 - \chi_{1,t+1}\right)\left(1 - \pi_{D,t+1}\right)\right)q_{t+1}^s, \\
x_{t+1}^p &= \chi_{2,t}x_{t+1}^h + \left(1 - \chi_{2,t}\right)x_{t+1}^l, \\
x_{t+1}^s &= f_t^h x_{t+1}^h \\
&\quad + f_t^{l,IR}x_{t+1}^l + \left(1 - f_t^h - f_t^{l,IR}\right)\left(r_{t+1}^l + \lambda q_{t+1}^s\right).
\end{aligned}
$$

The probability of defaults on implicit recourse in the next period conditional on the assets still bearing an implicit recourse

$$\pi_{D,t+1} = \left(1 - \chi_{D,t+1}\right)\left(1 - \sigma\right) + \chi_{D,t+1}.$$

The share of high quality assets on the resale markets

$$f_t^h = \frac{\chi_{1,t}\omega_t}{\chi_{1,t} + f_t^{NIR}\left(1 - \chi_{1,t}\right)\left(1 - \omega_t\right)}.$$

The share of low quality assets on the resale markets which bear implicit recourse

$$f_t^{l,IR} = \frac{\chi_{1,t}\left(1 - \omega_t\right)\left(1 - f_t^{NIR}\right)}{\chi_{1,t} + f_t^{NIR}\left(1 - \chi_{1,t}\right)\left(1 - \omega_t\right)}.$$

The share of low quality assets without implicit recourse

$$f_t^{NIR} = \left(1 - \chi_{D,t}\right)f_{t-1}^D + \chi_{D,t}.$$

The share of low quality assets with defaulted implicit recourse at the end of the period

$$f_t^D = \left(f_t^{NIR} + 1 - \sigma\right)\frac{\lambda K_t}{\lambda K_t + X_t}.$$

The costs of providing implicit recourse

$$P_t = \sigma\Lambda_{t,t+1}\left(1 - \chi_{D,t+1}\right)\left[A_{t+1}\left(\Delta_{t+1}^h - \Delta_{t+1}^l\right)K_{t+1}^{\alpha-1} + \lambda P_{t+1}\right].$$

The outstanding stock of implicit recourse obligations

$$SIR_t = \left(1 - f_t^{NIR}\right)\left(1 - \omega_t\right)K_t P_t.$$

When a firm defaults on the implicit recourse, the marginal value of its equity becomes ν_t^D, which is defined as

$$\nu_t^D = (1 - \sigma) + \sigma E_t \Lambda_{t,t+1} R_{t+1}^D \nu_{t+1}^D,$$

where R_{t+1}^D is the return on firm's equity when the firm looses the reputation and cannot sell on the securitization markets:

$$
\begin{aligned}
E_t R_{t+1}^D &= \pi \mu E_t \left(x_{t+1}^h \right) + (1 - \chi_s) \pi (1 - \mu) E_t \left(r_{t+1}^l + \lambda q_{t+1}^s \right) \\
&\quad + (\chi_s \pi (1 - \mu) + (1 - \pi)) E_t \frac{\left(x_{t+1}^p \right)}{q_t^p}.
\end{aligned}
$$

B.2   Markov regimes' properties

The Markov switching parameters $\bar{\chi}$ take the following values in different states. The parameter determining the share of investing firms $\chi_{1,t}$ takes the value π in a pooling equilibrium and $\pi\mu$ in a separating equilibrium, therefore, $\chi_1(1) = \pi$ and $\chi_1(2) = \chi_1(3) = \pi\mu$. The parameter determining the share of high quality assets available on the primary market $\chi_{2,t}$ takes the value 1 in a separating equilibrium and μ in a pooling equilibrium; therefore, $\chi_2(1) = \mu$ and $\chi_2(2) = \chi_2(3) = 1$. The parameter determining an economy-wide default on implicit recourse $\chi_{D,t}$ takes the value 0 in all non-default states and value 1 in the default state; therefore, $\chi_D(1) = \chi_D(2) = 0$ and $\chi_D(3) = 1$. The parameter determining the existence of a separating equilibria $\chi_{s,t}$ takes the value 1 in a separating equilibrium and 0 in a pooling equilibrium; therefore, $\chi_s(1) = 0$ and $\chi_s(2) = \chi_s(3) = 1$.

Parametrization of the model has to satisfy the endogenous conditions for the existence of a separating equilibrium or a pooling equilibrium and conditions for the equilibrium provision of implicit recourse and default on this recourse are satisfied according to the definitions of the 3 Markov regimes for a relevantly large subset of state variables combinations. These conditions are the following.

Pooling and separating equilibria conditions   The Markov Regime 1 with the high productivity and lowest dispersion should be a pooling equilibrium. Therefore, firms with access to low quality investment opportunities have to prefer mimicking firms with high quality investment opportunities rather than buying assets on the markets:

$$
\begin{aligned}
V_{i,t}\,(buying\,projects) &\leq V_{i,t}\,(mimicking\,\&\,no\,default)\ \forall i \in (\mathcal{L}_t \cap \mathcal{I}_t)\,, s_t = 1, \\
E_t \left(\Lambda_{t,t+1} R_{t+1}^n n_t \nu_{t+1} \right) \mid buying\,projects &\leq E_t \left(\Lambda_{t,t+1} R_{t+1}^n n_t \nu_{t+1} \right) \mid mimicking\,\&\,no\,default,
\end{aligned}
$$

$$
E_t \left(\Lambda_{t,t+1} \frac{x_{t+1}^h}{q_t^h} \right) \leq E_t \left(\Lambda_{t,t+1} \frac{x_{t+1}^l - (1 - \chi_{D,t+1}) \frac{1}{1-\theta} \left(\left(r_{t+1}^h - r_{t+1}^l \right) + \lambda P_{t+1} \right)}{\frac{1-\theta q_t^p}{1-\theta}} \right).
$$

While in Regime 2 and 3 the above inequality have to be exactly opposite for all firms with access to low quality investment opportunities

$$V_{i,t}\left(buying\,projects\right) \geq V_{i,t}\left(mimicking\,\&\,no\,default\right) \, \forall i \in \left(\mathcal{L}_t \cap \mathcal{I}_t\right), s_t \in \{2,3\}.$$

Default on implicit recourse conditions   For implicit recourse to have some value, it should not be defaulted upon at least in some states of the economy. According to the specification of the Markov regimes, any implicit recourse provided in a pooling equilibrium in Regime 1 should not be defaulted upon as long as the economy stays in Regime 1 or Regime 2. However, in Regime 3, all firms should find it optimal to default on the implicit recourse. For this to hold, we have to check the following conditions.

When the economy moves from Regime 1 to Regime 2, then for a significant subset of state variables, we should find in Regime 2 in period $t+1$ that all firms including those that had, in period t, access to low quality investment opportunities and which mimicked firms with high quality investment opportunities, will find it more profitable not to default on the existing implicit guarantees:

$$E_{t+1}V_{i,t+1}\left(not\,defaulting\right) \geq E_{t+1}V_{i,t+1}\left(defaulting\right) \, \forall i \in \left(\mathcal{L}_t \cap \mathcal{I}_t\right), s_t = 1, s_{t+1} = 2,$$

$$
\begin{aligned}
&(\frac{(1-\theta)\left(x_{t+1}^l - \frac{1}{1-\theta}\left(r_{t+1}^h - r_{t+1}^l + \lambda P_{t+1}\right)\right)}{1 - \theta q_t^p} \\
&- \frac{\left(1 - f_t^{NIR}\right)\left(1 - \omega_t\right)\lambda\left(r_{t+1}^h - r_{t+1}^l + \lambda P_{t+1}\right)}{R_t^N} + \frac{\left(1 - f_t^D\right)\left(1 - \omega_{t+1}\right)\lambda P_{t+1}}{R_{t+1}^N})\bar{\nu}_{t+1}^{ND} \quad (32) \\
&\geq \frac{(1-\theta)\left(r_{t+1}^l + \lambda q_{t+1}^s\right)}{1 - \theta q_t^{IR}}\bar{\nu}_{t+1}^D. \quad\quad\quad (33)
\end{aligned}
$$

This condition is sufficient to claim that implicit recourse is not defaulted upon for the respective subset of state variables as long as the economy stays in Regime 1, Regime 2 or Regime 3. This is because transferring from Regime 1 to Regime 2 implies the highest relative costs for honoring the implicit recourse.

Similarly, when an economy switches to Regime 3, all firms should find it optimal to default on their implicit guarantees. As discussed previously, due to the limited enforceability of the punishment rule, all firms find it optimal to default if a subset of firms default. Since Regime 3 can follow only after Regime 1, we again check the condition (32) but with an inverted inequality sign

$$E_{t+1}V_{i,t+1}\left(not\,defaulting\right) < E_{t+1}V_{i,t+1}\left(defaulting\right) \, \forall i \in \left(\mathcal{L}_t \cap \mathcal{I}_t\right), , s_t = 1, s_{t+1} = 3.$$

B.3 Equilibrium conditions for Markov-switching regimes

This section reviews the equilibrium conditions for the Markov-switching regimes in case when the government policy of asset purchases in the Deep Recession Regime is in place.

Government has to have a balanced budget every period, i.e.,

$$B_{t-1}\left(f_{t-1}^{Bh}r_t^h + \left(1 - f_{t-1}^{Bh}\right)r_t^l - r_t^h\right) + T_t = 0,$$

where f_{t-1}^{Bh} is the share of high quality assets in the asset purchase program and T_t is the aggregate tax revenue which is charged lump-sum to all financial firms.

Application of the government policy requires introduction of several other variables. I have already mentioned in the main text the new state variable, bonds as a share of capital f_t^B, whose law of motion is:

$$f_t^B = \left((1 - \chi_{D,t})f_{t-1}^B + \chi_{D,t}(1 - \omega_t)\right)\frac{\lambda K_t}{\lambda K_t + X_t}.$$

I also have to keep track of the share of high quality assets as a share of assets remaining on the balance sheets of the financial firms:

$$\omega_t^m = \frac{H_{t+1}^m}{K_{t+1}^m} = \frac{\lambda H_t}{\lambda\left(1 - f_{t-1}^B\right)K_t} = \frac{\omega_t}{1 - f_t^B}.$$

The investment function:

$$X_t = \frac{\chi_{1,t}(\sigma + \xi)\left[\left(\omega_t r_t^h + (1 - \omega_t)r_t^l + \lambda\left(1 - f_{t-1}^B\right)q_t^s + \lambda f_{t-1}^B q_t^B\right)K_t + Z_t\right]}{1 - \theta q_t^p}.$$

Consumption function:

$$\begin{aligned}C_t &= (1 - \sigma - \xi)\left[\left(\omega_t r_t^h + (1 - \omega_t)r_t^l\right)K_t \right.\\ &\quad \left.+ \lambda K_t\left(\left(1 - f_{t-1}^B\right)\left(\omega_t q_t^h + \left(1 - f_t^{NIR}\right)(1 - \omega_t)q_t^l + f_t^{NIR}(1 - \omega_t)q_t^s\right) + f_{t-1}^B q_t^B\right) + Z_t\right].\end{aligned}$$

The consumer goods market clearing condition:

$$\left(\omega_t r_t^h + (1 - \omega_t)r_t^l\right)K_t + Z_t = X_t + C_t + Z_{t+1}.$$

Law of motion for capital

$$K_{t+1} = \lambda K_t + X_t,$$

and law of motion for the share of high quality assets

$$\omega_{t+1} = \frac{H_{t+1}}{K_{t+1}} = \frac{\lambda H_t + X_t^H}{\lambda K_t + X_t} = \frac{\omega_t \lambda K_t + \chi_{2,t}X_t}{\lambda K_t + X_t} = \frac{\omega_t\lambda + \chi_{2,t}X_t/K_t}{\lambda + X_t/K_t}.$$

Capital goods market clearing conditions: I calibrate the model such that there is a

positive amount of storage as well as investment in the economy. Since the return on storage is $R_{t+1}^Z = 1$, the market clearing conditions are the following:

$$E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^h}{q_t^h}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^l}{q_t^l}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^p}{q_t^p}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^s}{q_t^s}\right] = E_t\left[\Lambda_{t,t+1}\frac{x_{t+1}^B}{q_t^B}\right] = E_t\left[\Lambda_{t,t+1}\right],$$

where the next period cash flows and values of these assets are defined

$$
\begin{aligned}
x_{t+1}^h &= r_{t+1}^h + \lambda\left(1 - \chi_{1,t+1}\right)q_{t+1}^h + \lambda\chi_{1,t+1}q_{t+1}^s, \\
x_{t+1}^l &= \pi_{D,t+1}r_{t+1}^l + \left(1 - \pi_{D,t+1}\right)r_{t+1}^h \\
&\quad + \lambda\left(1 - \chi_{1,t+1}\right)\left(1 - \pi_{D,t+1}\right)q_{t+1}^l + \lambda\left(1 - \left(1 - \chi_{1,t+1}\right)\left(1 - \pi_{D,t+1}\right)\right)q_{t+1}^s, \\
x_{t+1}^p &= \chi_{2,t}x_{t+1}^h + \left(1 - \chi_{2,t}\right)x_{t+1}^l, \\
x_{t+1}^s &= \chi_{D,t}\left(r_{t+1}^B + \lambda q_{t+1}^B\right) + \left(1 - \chi_{D,t}\right)\left[f_t^h x_{t+1}^h \right.\\
&\quad \left.+ f_t^{l,IR}x_{t+1}^l + \left(1 - f_t^h - f_t^{l,IR}\right)\left(r_{t+1}^l + \lambda q_{t+1}^s\right)\right].
\end{aligned}
$$

The probability of defaults on implicit recourse in the next period conditional on the assets still bearing an implicit recourse

$$\pi_{D,t+1} = \left(1 - \chi_{D,t+1}\right)\left(1 - \sigma\right) + \chi_{D,t+1}.$$

The share of high quality assets on the resale markets

$$f_t^h = \frac{\chi_{1,t}\omega_t^m}{\chi_{1,t} + f_t^{NIR}\left(1 - \chi_{1,t}\right)\left(1 - \omega_t^m\right)}.$$

The share of low quality assets on the resale markets which bear implicit recourse

$$f_t^{l,IR} = \frac{\chi_{1,t}\left(1 - \omega_t^m\right)\left(1 - f_t^{NIR}\right)}{\chi_{1,t} + f_t^{NIR}\left(1 - \chi_{1,t}\right)\left(1 - \omega_t^m\right)}.$$

The share of low quality assets without implicit recourse on the balance sheets of financial firms

$$f_t^{NIR} = \left(1 - \chi_{D,t}\right)f_{t-1}^D.$$

The share of low quality assets with defaulted implicit recourse at the end of the period on the balance sheets of financial firms

$$f_t^D = \left(f_t^{NIR} + 1 - \sigma\right)\frac{\lambda K_t}{\lambda K_t + X_t}.$$

The costs of providing implicit recourse

$$P_t = \sigma\Lambda_{t,t+1}\left(1 - \chi_{D,t+1}\right)\left[A_{t+1}\left(\Delta_{t+1}^h - \Delta_{t+1}^l\right)K_{t+1}^{\alpha-1} + \lambda P_{t+1}\right].$$

The outstanding stock of implicit recourse obligations

$$SIR_t = \left(1 - f_t^{NIR}\right)\left(1 - \omega_t\right) K_t P_t.$$

When a firm defaults on the implicit recourse, the marginal value of its equity becomes ν_t^D, which is defined as

$$\nu_t^D \;=\; (1 - \sigma) + \sigma E_t \Lambda_{t,t+1} R_{t+1}^D \nu_{t+1}^D,$$

where R_{t+1}^D is the return on firm's equity when the firm looses the reputation and cannot sell on the securitization markets:

$$\begin{aligned}
E_t R_{t+1}^D \;=\;& \pi \mu E_t\left(x_{t+1}^h\right) + (1 - \chi_s)\,\pi\,(1 - \mu)\,E_t\left(r_{t+1}^l + \lambda q_{t+1}^s\right) \\
&+ (\chi_s \pi\,(1 - \mu) + (1 - \pi)) E_t \frac{\left(x_{t+1}^p\right)}{q_t^p}.
\end{aligned}$$