Appendix (for online publication)

A Proofs for the simple model

A.1 Optimal choice of recourse under symmetric information

Here I derive the optimal choice of recourse when the asymmetric information friction is absent by assumption or when the equilibrium is separating, i.e., condition (3.7) is satisfied on the interval, where the recourse is chosen.

No binding SGC. In this case the quantity of sold assets $s_i x_i$ is demand determined, i.e., independent of recourse choice, in equilibrium. The optimal choice of explicit recourse solves:

$$\frac{\partial w_i}{\partial r_i^{EG}} = \frac{\partial[r^h\left(n_i + (q_i - 1)\right) - \max\left\{r^{EG} - r^h, 0\right\}(1+\tau)]s_i x_i}{\partial r_i^{EG}} = \left(r^h \frac{\partial q_i}{\partial r_i^{EG}} - 1 - \tau\right) s_i x_i = 0.$$

Using the market clearing condition $q_i = r^{EG} q^h / r^h$, the above condition collapses to $q^h = 1 + \tau$. This implies that only when $q^h \geq 1 + \tau > 1$ firms provide explicit recourse. Similarly the optimal choice of implicit recourse conditional on being credible (i.e. satisfying the non-default condition 3.6) solves:

$$\frac{\partial w_i}{\partial r_i^{IG}} = \frac{\partial[r^h\left(n_i + (q_i - 1)\right) - \max\left\{\chi_i r^{IG} - \max\left\{r^{EG}, r^h\right\}, 0\right\}]s_i x_i}{\partial r_i^{IG}} = \left(r^h \frac{\partial q_i}{\partial r_i^{IG}} - 1\right) s_i x_i = 0,$$

which collapses to $q^h = 1$. But when $q^h = 1$, securitization is not profitable, which implies that no credible implicit recourse exceeding a project return can be provided. As a result the equilibrium level of implicit recourse is given by the maximum recourse satisfying condition (3.6) with equality. Therefore, when the SGC is not binding (i.e., when $q^h = 1$), neither explicit nor implicit recourse exceeding the project return is provided.

Binding SGC. In this case the quantity of sold assets is supply determined $s_i x_i = \theta x_i$, i.e., it depends on the resources of the investing firm, and therefore also on $q_i(r_i^{EG})$. The optimal choice of explicit recourse then solves:

$$\frac{\partial w_i}{\partial r_i^{EG}} = \frac{\partial}{\partial r_i^{EG}} \frac{r^h(1-\theta) - \theta \max\left\{r_i^{EG} - r^h, 0\right\}(1+\tau)}{1 - \theta q_i} n_i = 0,$$

which collapses to

$$q^h = (1+\tau)\,/\,(1+\theta\tau)\,. \tag{A.1}$$

In equilibrium the recourse provision lowers the price of high quality assets without guarantee $q^h = \frac{r^h}{r^{EG}} q^G = \frac{r^h}{r^{EG}} \frac{1-\pi\mu}{\theta}$, which brings the price to the equilibrium level given by (A.1). Similarly, the optimal choice of implicit recourse conditional on being credible (i.e., satisfying

3.6) solves:

$$\frac{\partial w_i}{\partial r_i^{IG}} = \frac{\partial}{\partial r_i^{IG}} \frac{r^h (1-\theta) - \theta \max\left\{\chi_i r^{IG} - \max\left\{r^{EG}, r^h\right\}, 0\right\}(1+\tau)}{1-\theta q_i} n_i = 0,$$

which again collapses to $q^h = 1$. But since under $q^h = 1$ securitization is not profitable, the equilibrium level of implicit recourse is again given by the maximum recourse satisfying the non-default condition (3.6) with equality.

Finally, when the implicit recourse is provided, the explicit recourse will not be provided if

$$\frac{\partial w_i}{\partial r_i^{EG}} = \frac{\partial}{\partial r_i^{EG}} \frac{r^h (1-\theta) - \theta \max\left\{r_i^{EG} + \Delta^{IG} - r^h, 0\right\} - \theta \max\left\{r_i^{EG} - r^h, 0\right\}\tau}{1-\theta q_i} n_i < 0,$$

where $\Delta^{IG} = \max\{\chi r^{IG} - r^E, 0\}$. When $r^{EG} = 0$, this condition collapses to

$$q^h < \left(1 + \tau + d\Delta^{IG}/dr^{EG}\right) / \left(1 + P\theta\tau + d\Delta^{IG}/dr^{EG}\right), \tag{A.2}$$

where $P = r^{IG}/r^h$ is the price premium for implicit recourse.

A.2 Proof of Claim 2

Full characterization of the equilibrium. Suppose condition (3.10) in Proposition 2 is satisfied and $\tau > \frac{1}{\theta}\left(\frac{1-\theta}{\pi\mu} - 1\right)$. Then the equilibrium is characterized by one of the following cases:[1]

1. Separating equilibrium (Case H): Only firms with high-quality projects invest (takes place if $q^l < \frac{\pi\mu}{1-\theta} < 1$)

2. Pooling equilibrium: firms with both high- and low-quality projects invest:

   - Case M: Firms with low-quality projects invest with probability ϕ (takes place if $\frac{\pi\mu}{1-\theta} \le q^l \le \frac{\pi}{1-\theta}$);
   - Case B: All firms with low-quality projects invest and securitize project returns (takes place if $q^l > \frac{\pi}{1-\theta}$)

Case H takes place when firms with low-quality assets refrain from investing, i.e., condition (3.7) is satisfied, which under asymmetric information becomes:

$$\frac{r^h}{q^h} > \frac{(1-\theta)\, r^l}{1-\theta q^h}. \tag{A.3}$$

[1]Note that prices q^l, q^l under asymmetric information are market prices of assets of known quality with returns r^h and r^l, respectively. I drop subscript i because the equilibrium is symmetrical, meaning that firms with the same projects choose the same controls.

Using the market clearing condition $r^h/q^h = r^l/q^l$ and substituting the equilibrium price q^h from eq. (3.9), I can rewrite (A.3) as:

$$q^l < \frac{1-\theta q^h}{1-\theta} = \frac{\pi\mu}{1-\theta} < 1. \tag{A.4}$$

or equivalently as minimum requirement on the return dispersion:

$$\frac{r^h}{r^l} > q^h \frac{1-\theta}{1-\theta q^h} = \frac{(1-\theta)(1-\pi\mu)}{\pi\mu\theta}.$$

The optimal choice of explicit recourse is chosen by (A.1) which implies that recourse is not provided if $q^h < (1+\tau)/(1+\theta\tau)$, or equivalently if $\tau > \frac{1}{\theta}\left(\frac{1-\theta}{\pi\mu} - 1\right) \equiv \bar{\tau}$ (obtained after substituting for q^h from eq. 3.9).

Case M takes place when firms with low-quality assets are mixing, i.e., they are indifferent about buying assets in the market and investing: $r^h/q^h = (1-\theta) r^l / (1-\theta q)$, where price q reflects the average asset quality bringing average return $\mu r^h + \phi (1-\mu) r^l$. The indifference condition can be rewritten as follows: $q^l = (1-\theta q) / (1-\theta) = \pi(\mu + (1-\mu)\phi)/(1-\theta)$.

Case B takes place when both firms with high- and low- quality projects invest and securitize to the maximum capacity, i.e., when

$$q^l > \frac{1-\theta q}{1-\theta} = \frac{\pi}{1-\theta},$$

where the second equality follows from $q = \frac{1-\pi}{\theta}$, which can be obtained by the combination of aggregate investment equations:

$$X^h = \frac{\pi\mu N}{1-\theta q}; \quad X^l = \frac{\pi(1-\mu)N}{1-\theta q}$$

with the goods market clearing condition $N = X^h + X^l$.

A.3 Proof of Claim 4 and Proposition 3

Suppose condition (3.10) in Proposition 2 is satisfied and $\tau < \bar{\tau}$. Then the equilibrium is separating if firms with low-quality assets refrain from investing. In other words, condition (3.7) is satisfied, which in this case becomes:

$$\frac{r^h}{q^h} > \frac{(1-\theta) r^l - \theta\left(r^{EG} - r^l\right)(1+\tau)}{1-\theta q^G}, \tag{A.5}$$

where q^G is the market price for assets with a guarantee. Using the asset market clearing condition $r^h/q^h = r^l/q^l$ and substituting q^G from (3.9) and q^h from (A.1), I can rewrite (A.5) as:

$$\frac{r^h}{r^l} > \frac{1+\tau}{1+\tau-\pi\mu\tau}.$$

It remains to prove that condition (A.5) is satisfied for larger subset of parameters than condition (A.3). This is the case because the left-hand side (LHS) of (A.5) is larger than (A.3), given that $q^h \mid_{r^{EG}=0} = (1-\pi\mu)/\theta > q^h \mid_{r^{EG}>r^h} = r^h q^G/r^G = (1-\pi\mu)r^h/(\theta r^{EG})$. Moreover, while the denominator on the RHS is the same in both conditions, the numerator on the RHS is smaller in condition (A.5) because $r^{EG} > r^h$.

A.4 Proof of Claim 5

I first define all pooling and separating PBE and then refine them using the intuitive criterion. Finally, I derive conditions for the existence of the separating equilibrium. Note that I focus on cases where explicit recourse is not provided, i.e. when $\tau < \tilde{\tau} = \frac{1+d\Delta^{IR}/dr^{EG}}{P\theta}\left(\frac{1-\theta P}{\pi\mu} - 1\right)$, a condition that can be obtained by combining (A.2) with $q^h = q^G/P = (1-\pi\mu)/(P\theta)$.

Pooling PBE. In order to identify pooling equilibria, it is useful to establish several points. Firstly, analogously to Claim 1, there is no pooling equilibrium in cases where all investing firms choose a level of recourse lower than the maximum recourse on which firms selling low-quality assets would not default $r^{IG} \leq r^{IG}_{l,cred,p}$.[2] Because, due to competition, firms with both high- and low-quality projects increase implicit recourse to $r^{IG}_{l,cred,p}$. The beliefs of saving firms do not affect the asset price since both types honor the implicit recourse on this interval.

Secondly, when recourse exceeds $r^{IG}_{l,cred,p}$, firms selling low-quality assets default on the recourse and the asset owner receives $r^l < r^{IG}_{l,cred,p}$. This negatively affects the market price and disincentivizes increasing the recourse above $r^{IG}_{l,cred,p}$.

Thirdly, there are two natural upper limits for the implicit recourse in a pooling equilibrium. The first is the maximum credible recourse level that a firm selling high-quality assets can provide: $r^{IG}_{h,cred,p}$.[3] The second limit is the level of recourse at which all low quality firms prefer to separate r^{IG}_{minsep}.[4]

[2] At the recourse $r^{IG}_{l,cred,p}$ in the pooling equilibrium, firms selling low-quality assets are indifferent between defaulting and honoring the recourse: $\nu^{ND} w_i \mid_{r_i^{IG}=r^{IG}_{l,cred,p}} = \nu^D w_i \mid_{r_i^{IG}=r^{IG}_{l,cred,p}} \quad \forall i \in \mathcal{L}$.

[3] At recourse $r^{IG}_{h,cred,p}$ in the pooling equilibrium, firms selling high-quality assets are indifferent between defaulting and honoring the recourse: $\nu^{ND} w_i \mid_{r_i^{IG}=r^{IG}_{h,cred,p}} = \nu^D w_i \mid_{r_i^{IG}=r^{IG}_{h,cred,p}} \quad \forall i \in \mathcal{H}_1$.

[4] At recourse r^{IG}_{minsep}, only firms with high-quality projects invest. All firms with low-quality projects choose not to invest and are marginally indifferent between buying assets in the market and mimicking firms

The above-mentioned considerations lead to the definition of the whole set of pooling PBE, which consist of three types.

Case B***:*** Firms with high- and low-quality projects choose recourse $r^{IG*} = r^{IG}_{l,cred,p}$. Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following: $\varphi_{i,1} \mid_{r_i^{IG} > r^{IG}_{l,cred,p}} = 0$ and unrestricted for intervals $0 < r_i^{IG} < r^{IG}_{l,cred,p}$. In this equilibrium, no firm defaults on an implicit recourse.

Case B'***:*** Firms with high and low-quality projects select $r_i^{IG} = r^{IG*}$ such that:

$$r^{IG}_{lb,p} \leq r^{IG*} \leq \min\left\{r^{IG}_{minmix}, r^{IG}_{h,cred,p}\right\}.$$

Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the following: $\varphi_i \mid_{r_i^{IG} > r^{IG*}} = 0$, and $\varphi_i \mid_{r_i^{IG} < r^{IG*}} \leq \mu$.[5]

Case M***:*** Firms with high-quality projects and a fraction ϕ of firms with low-quality projects invest and choose $r_i^{IG} = r^{IG*}$ such that:

$$\min\left\{r^{IG}_{minmix}, r^{IG}_{h,cred,p}\right\} < r^{IG*} < \min\left\{r^{IG}_{minsep}, r^{IG}_{h,cred,p}\right\}.$$

Saving firms' out-of-equilibrium beliefs that sustain this equilibrium can be the same as in the Case $B_1^{A\prime}$.

Separating PBE. There is potentially a continuum of separating PBE, where firms with low-quality projects save and buy securitized assets from firms with high-quality projects. Firms with high-quality projects invest, securitize, and provide implicit recourse $r^{IG*} \in \left[r^{IG}_{minsep}, r^{IG}_{h,cred,s}\right]$, where $r^{IG}_{h,cred,s}$ is the maximum implicit recourse that firms selling high-quality assets can credibly provide in a separating equilibrium.

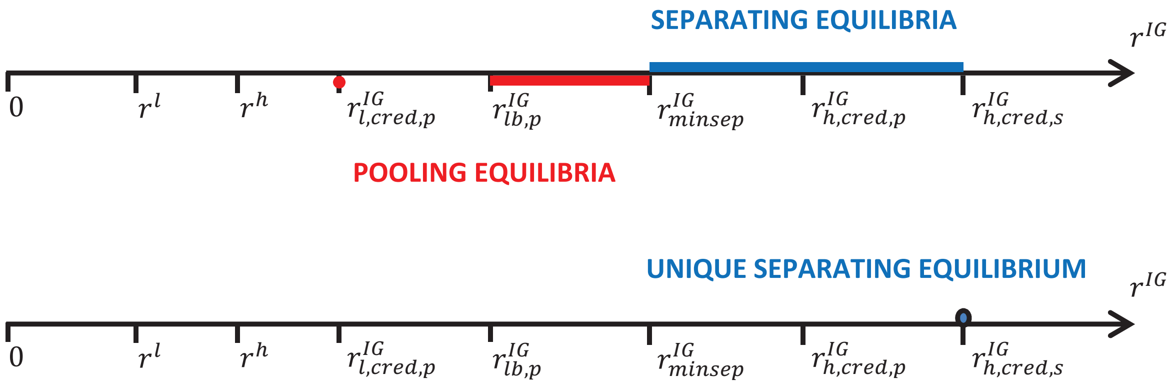
Saving firms' out-of-equilibrium beliefs that sustain this equilibrium are for instance the following: $\varphi_i \mid_{r_i^{IG} > r^{IG*}} = 0$ and unrestricted for $r_i^{IG} < r^{IG*}$.

Application of the intuitive criterion: If any separating equilibrium exists, then all pooling equilibria are dominated, and therefore, fail the intuitive criterion (Cho and Kreps, 1987). Due to competition, the intuitive criterion selects the separating equilibrium

with high-quality projects: $\nu^{ND} w_i \mid_{x_i = 0} = \nu^D w_i \mid_{x_i > 0, \varphi_{i,1} = 1, r_i^{IG} = r^{IG}_{minsep}} \ \forall i \in \mathcal{L}_1$.

[5] r^{IG}_{minmix} is the recourse in an equilibrium, where all firms with high- and low-quality projects invest, securitize and provide this recourse, but firms with low-quality projects are marginally indifferent between this strategy and buying assets in the market. $r^{IG}_{lb,p}$ is the recourse in an equilibrium, where all firms with high- and low-quality projects invest, securitize and provide this recourse, but firms with high-quality projects are marginally indifferent between this strategy and deviating to provision of $r^{IG}_{l,cred,p}$ even if this implies $\varphi_i = 0$. There is no pooling equilibrium with $r^{IG}_{l,cred,p} < r^{IG} < r^{IG}_{lb,p}$, since both types have incentives to decrease implicit recourse to $r_j^{IG} = r^{IG}_{l,cred,p}$. This is because the negative price effect of equilibrium defaults on recourse by sellers of low-quality projects, together with additional costs of the higher implicit recourse, outweighs the positive price effect of the higher implicit recourse.

Figure A.1. Intuitive criterion selects a unique separating equilibrium



Note: On the upper axis I highlight the implicit recourse level for a continuum of separating and pooling PBE in blue and red, respectively. The intuitive criterion selects a unique separating equilibrium with $r_i^{IG} = r^{IG}_{h,cred,s}$ $\forall i \in \mathcal{H}_1$ (on the lower axis).

with maximum credible implicit recourse $r^{IG*} = r^{IG}_{h,cred,s}$ (see Figure A.1). In cases where no separating PBE exists, application of the intuitive criterion does not affect potential multiplicity of pooling equilibria. However, for a subset of parameters there is a unique equilibrium (see Figure A.2).

The **unique separating equilibrium** takes place when condition (3.7) is satisfied, which becomes:

$$\frac{r^h}{q^h}\nu^{ND} > \frac{r^l}{\frac{1-\theta q^G}{1-\theta}}\nu^D. \tag{A.6}$$

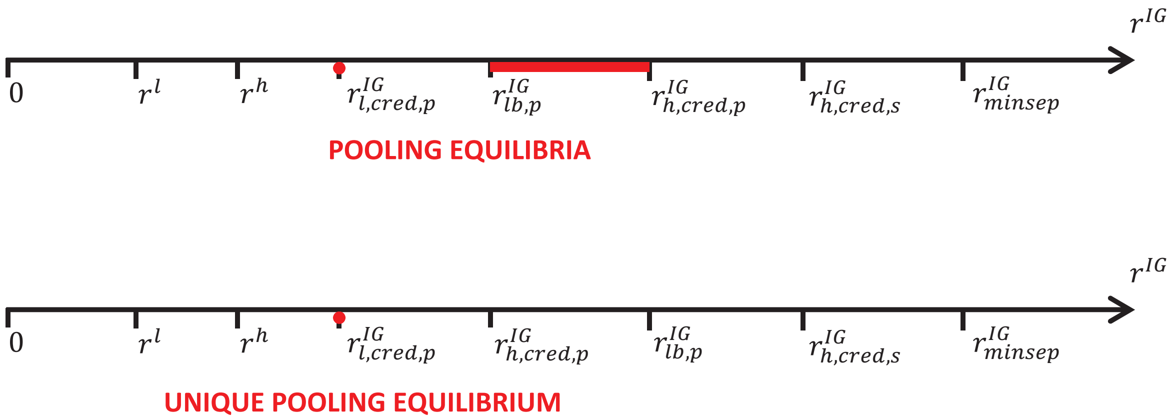
This condition states that returns from buying assets multiplied by the continuation value conditional on honoring recourse has to exceed the return from mimicking firms with high-quality projects and defaulting on the recourse multiplied by the lower continuation value ν^D due to default. I use the condition for the maximum credible implicit recourse satisfying the non-default condition (3.6), which can be written as $(1-\theta P)\,\nu^{ND} = (1-\theta)\,\nu_D$,[6] in order to rearrange condition (A.6):

$$\frac{r^h}{r^l} > \frac{(1-\theta P)\,q^h}{1-\theta P q^h} = \frac{(1-\theta P)\,(1-\pi\mu)}{P\pi\mu\theta}. \tag{A.7}$$

A **unique pooling equilibrium** takes place when only Case B is an equilibrium, i.e.

[6]Note that in the separating equilibrium, selected by the intuitive criterion, mimicking firms with low-quality projects choose to default on implicit recourse.

Figure A.2. Separating equilibrium does not exist



Note: The minimum recourse level needed for separation r^{IG}_{minsep} exceeds the maximum level of recourse that can be credibly provided without default $r^{IG}_{h,cred,s}$. Therefore, no separating equilibrium exists and the intuitive criterion does not eliminate any PBE. There remains a multiplicity of pooling equilibria (upper panel) or a unique pooling equilibrium with $r^{IG} = r^{IG}_{l,cred,p}$ (lower panel). The latter takes place because $r^{IG}_{lb,p}$ exceeds the maximum credible recourse provided in a pooling equilibrium $r^{IG}_{h,cred,p}$.

when

1. $r^{IG}_{lb,p}$ satisfying $r^{IG}_{lb,p} > r^{IG}_{l,cred,p}$ exceeds the maximum credible level of recourse that can be provided by firms with high-quality projects: $r^{IG}_{lb,p} > r^{IG}_{h,cred,p}$; or when

2. there is no recourse satisfying all properties for $r^{IG}_{lb,p}$. Recall that $r^{IG}_{lb,p} > r^{IG}_{l,cred,p}$ is the equilibrium recourse level at which firms with high-quality projects are indifferent between providing such recourse or deviating from the equilibrium by providing a recourse $r^{IG}_{l,cred,p}$. As I show below for sufficiently small rates of occurrence of high-quality projects (μ), there are no $r^{IG}_{lb,p} > r^{IG}_{l,cred,p}$ that would satisfy this indifference condition.

Firms with high-quality projects prefer to deviate to $r^{IG}_{l,cred,p}$ if

$$
\begin{aligned}
V_i \,|_{r^{IG}=r^{IG}_{l,cred,p}+\Delta} &< V_i \,|_{r^{IG}=r^{IG}_{l,cred,p}} \\
\frac{r^h - \theta\left(r^{IG}_{l,cred,p} + \Delta\right)}{1-\theta\left(\mu \frac{r^{IG}_{l,cred,p}+\Delta}{r^h} + (1-\mu)\frac{r^l}{r^h}\right) q^h} &< \frac{r^h - \theta r^{IG}_{l,cred,p}}{1-\theta \frac{r^{IG}_{l,cred,p}}{r^h} q^h}
\end{aligned}
\tag{A.8}
$$

Condition A.8 holds for all parameters satisfying the following, more restrictive, condition:

$$\frac{r^h - \theta\left(r^{IG}_{l,cred,p} + \Delta\right)}{1 - \theta\left(\mu \frac{r^{IG}_{l,cred,p}+\Delta}{r^h} + (1-\mu)\frac{r^l}{r^h}\right) q^h} < \frac{r^h - \theta r^{IG}_{l,cred,p}}{1 - \theta\left(\mu \frac{r^{IG}_{l,cred,p}}{r^h} + (1-\mu)\frac{r^l}{r^h}\right) q^h},$$

which can be simplified to

$$\begin{aligned}
\Delta\theta\left(1 - \theta q^h\left(\mu \frac{r^{IG}_{l,cred,p}}{r^h} + (1-\mu)\frac{r^l}{r^h}\right)\right) &> \Delta\theta\mu q^h\left(1 - \theta\frac{r^{IG}_{l,cred,p}}{r^h}\right) \\
\mu &< \frac{1-\theta q^l}{q^h - q^l\theta}.
\end{aligned}$$

Intuitively, for low occurrence of high-quality projects μ, the negative price effect of defaults by firms with low-quality projects dominates the positive effects of a higher guarantee.

A.5 Endogenizing the "skin in the game"

Consider a simple moral hazard problem, which endogenizes the existence of the SGC. Firms can divert investment funds, but still issue and sell assets backed by returns from non-existing projects. This can be observed only in period 2. But before selling securitized assets to buyers, the issuing firm can be asked to prove that it has spent a fraction of the intended investment from its own resources. The investment retained by the originator is verifiable. Therefore, this moral hazard problem can be eliminated if investors require the issuing firms to retain a sufficiently large "skin in the game," which would satisfy the incentive compatible constraint (ICC):

$$V\mid_{diverting\,investment\,funds} \leq V\mid_{investing\,properly}.$$

When a firm diverts funds, its return on endowment is $R\mid_{diverting\,investment\,funds} = \left(\frac{\theta q_i}{(1-\theta_i)}\right)^{\iota_i}$, because it has to spend $1-\theta_i$ per unit of investment from its own resources, which cannot be recovered, and it receives $\theta_i q_i$ per unit of investment. ι_i is the number of times the firm reuses the returns from this operation to repeat this fund diversion scheme. Since there is no restriction of sequential asset issuance (a practice using the funds from the asset sale to cover the "skin in the game" fraction of a new investment within the same period), then for an infinitely small firm ι_i is unbounded. As a result, the ICC will always fails unless $\theta_i q_i \leq (1-\theta_i)$, or equivalently unless

$$\theta_i \leq \frac{1}{q_i + 1}. \tag{A.9}$$

Intuitively, the higher the sale price of loans q_i, the larger "skin in the game" $(1-\theta_i)$ is required to prevent the moral hazard problem. Such SGC becomes binding when $\pi\mu < 1/2$, then the price of loans is equal to $q_i = \frac{1-\pi\mu}{\pi\mu}$ (obtained from combining 3.9 with a binding A.9).

B Derivation of firms' policy functions in the full model

I characterize the recursive equilibrium (I drop the time subscripts and use $\prime$ to denote next-period variables).

Definition 3. A recursive competitive perfect Bayesian equilibrium consists of prices $\{q^h\left(\bar{\mathbf{S}}_t\right)$, $q^l\left(\bar{\mathbf{S}}_t\right)$, $\{q_j^P(\bar{\mathbf{S}}_t)\}$, $\{q_j^S(\bar{\mathbf{S}}_t)\}\}$ $\forall j$, gross profits per unit of capital $\left\{r^h(\bar{\mathbf{S}}_t), r^l(\bar{\mathbf{S}}_t)\right\}$, individual decision rules $\{c(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $x(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $h^O(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $l^O(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $r^{EG\prime}(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $r^{IG\prime}(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t)$, $\{a_j^P(\bar{\mathbf{s}}_{i,t}, b_{j,t}, \varphi_j(b_{j,t});\bar{\mathbf{S}}_t)\}$, $\{a_j^S(\bar{\mathbf{s}}_{i,t}, \varphi_j^{\mathbf{S}}(b_{j,t});\bar{\mathbf{S}}_t)\}$, $\chi(\bar{\mathbf{s}}_{i,t}^e;\bar{\mathbf{S}}_t)\}$ $\forall j$, value functions $\{V^{ND}\left(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t\right)$, $V^D\left(\bar{\mathbf{s}}_{i,t};\bar{\mathbf{S}}_t\right)\}$, and the law of motion for $\bar{\mathbf{S}}_t = \{K_t, \omega_t, A_t, \Sigma_t\}$ such that:

(i) individual decision rules and value functions solve firms' problems given their beliefs taking prices, gross profits per unit of capital, and law of motion for $\bar{\mathbf{S}}_t$ as given;

(ii) both asset and goods markets clear;

(iii) firms update their beliefs using the Bayes' rule on the equilibrium path based on other firms observable controls $\{\varphi_j(b_{j,t})\}$ $\forall j$; and

(iv) the law of motion for $\bar{\mathbf{S}}$ is consistent with the individual firm's decisions.

The set of individual state variables early in the period t is $\bar{\mathbf{s}}_{i,t}^e = \{x_{i,t-1}, \{a_{i,j,t-1}^P\}, \{a_{i,j,t-1}^S\}, h_{i,t-1}^O, l_{i,t}^O, r_{i,t}^{EG}, r_{i,t}^{IG}, \sigma_{i,t-1}\}$ $\forall j$ and late in the period is $\bar{\mathbf{s}}_{i,t} = \{\bar{\mathbf{s}}_{i,t}^e, \kappa_{i,t}, \chi_{i,t}\}$.
From the FOCs, I can obtain the following Euler equations in cases where the SGC is binding for all investing firms:

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{\hat{r}_{j,t+1}+\lambda q_{j,t+1}^S}{q_{j,t}^P}\right] = 1\ \forall i\in\mathcal{S}_t, \forall j\in\mathcal{I}_t, \tag{B.1}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{j,t+1}+\lambda\left(\varphi_{j,t}^S q_{t+1}^h+\left(1-\varphi_{j,t}^S\right)q_{t+1}^l\right)}{q_{j,t}^S}\right] = 1\ \forall i\in\mathcal{S}_t, \forall j\in\mathcal{I}_{t-1}, \tag{B.2}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^h+\lambda q_{t+1}^h}{q_t^h}\right] = 1\ \forall i\in\mathcal{S}_t, \tag{B.3}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{r_{t+1}^l+\lambda q_{t+1}^l}{q_t^l}\right] = 1\ \forall i\in\mathcal{S}_t, \tag{B.4}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{(1-\theta)\left(r_{t+1}^h+\lambda q_{i,t+1}^S\right)-\theta g_{i,t+1}}{1-\theta q_{i,t}^P}\right] = 1\ \forall i\in\mathcal{H}_t\cap\mathcal{I}_t, \tag{B.5}$$

$$E_t\left[\beta\frac{c_{i,t}}{c_{i,t+1}}\frac{(1-\theta)\left(r_{t+1}^l+\lambda q_{i,t+1}^S\right)-\theta g_{i,t+1}}{1-\theta q_{i,t}^P}\right] = 1\ \forall i\in\mathcal{L}_t\cap\mathcal{I}_t, \tag{B.6}$$

where $\varphi^S_{j,t}$ is the belief of firms in period t that the asset issued by firm j in period $t-1$ is of high quality. Due to the logarithmic utility function, all firms consume a $(1-\beta)$ fraction of their wealth:

$$c_{i,t} = (1-\beta)\, w_{i,t}\ \forall i, t. \tag{B.7}$$

Under binding SGC, investing firms $(\mathcal{I}_t)$ invest all of the unconsumed part of their wealth into new projects and sell the maximum fraction of investment θ to saving firms:[7]

$$a^P_{i,i,t} = \frac{\beta w_{i,t}}{\frac{\left(1-\theta q^P_{i,t}\right)}{(1-\theta)}}\ \forall i \in \mathcal{I}_t,$$

Saving firms $\mathcal{S}_t$ are, in equilibrium, indifferent about investing in different assets. All of them try to diversify their investment, so I guess and verify that, in equilibrium, all will allocate the same fraction of wealth into different assets:

$$\begin{aligned}
q^P_{j,t} a^P_{i,j,t} &= \zeta^{hP}_j \beta w_{i,t}\ \forall i \in \mathcal{S}_t\ \forall j \in \mathcal{H}_t \cap \mathcal{I}_t,\\
q^P_{j,t} a^P_{i,j,t} &= \zeta^{lP}_j \beta w_{i,t}\ \forall i \in \mathcal{S}_t\ \forall j \in \mathcal{L}_t \cap \mathcal{I}_t,\\
q^S_{j,t} a^S_{i,j,t} &= \zeta^{hS}_j \beta w_{i,t}\ \forall i \in \mathcal{S}_t\ \forall j \in \mathcal{H}_{t-1} \cap \mathcal{I}_{t-1},\\
q^S_{j,t} a^S_{i,j,t} &= \zeta^{lS}_j \beta w_{i,t}\ \forall i \in \mathcal{S}_t\ \forall j \in \mathcal{L}_{t-1} \cap \mathcal{I}_{t-1},\\
q^h_t h^O_{i,t} &= \zeta^{hO} \beta w_{i,t}\ \forall i \in \mathcal{S}_t,\\
q^l_t l^O_{i,t} &= \zeta^{lO} \beta w_{i,t}\ \forall i \in \mathcal{S}_t.
\end{aligned}$$

Wealth fractions sum up to one: $\sum_{j\in\mathcal{H}_t\cap\mathcal{I}_t} \zeta^{hP}_j + \sum_{j\in\mathcal{H}_t\cap\mathcal{I}_t} \zeta^{lP}_j + \sum_{j\in\mathcal{H}_{t-1}\cap\mathcal{I}_{t-1}} \zeta^{hS}_j + \sum_{j\in\mathcal{L}_{t-1}\cap\mathcal{I}_{t-1}} \zeta^{lS}_j + \zeta^{hO} + \zeta^{lO} = 1$. Since firms' decision rules are either independent or linear in wealth, we do not have to keep track of the wealth distribution Σ_t.

The consumption of firms in the following period depends on the return from their investment:

$$\begin{aligned}
c_{i,t+1} &= (1-\beta)\, [\sum_{j\in\mathcal{I}_t} a^P_{i,j,t}\left(\hat{r}_{j,t+1} + \lambda q^S_{j,t+1}\right) + \sum_{j\in\mathcal{H}_{t-1}\cap\mathcal{I}_{t-1}} a^S_{i,j,t}\left(r^h_{t+1} + \lambda q^h_{t+1}\right)\\
&\quad + \sum_{j\in\mathcal{L}_{t-1}\cap\mathcal{I}_{t-1}} a^S_{i,j,t}\left(r^l_{t+1} + \lambda q^l_{t+1}\right) + h^O_{i,t}\left(r^h_{t+1} + \lambda q^h_{t+1}\right) + l^O_{i,t}\left(r^l_{t+1} + \lambda q^l_{t+1}\right)]\ \forall i \in \mathcal{S}_t,\\
c_{i,t+1} &= (1-\beta)\, a_{i,i,t}\left(r^h_{t+1} + \lambda q^S_{i,t+1} - \frac{\theta}{(1-\theta)} g^T_{i,t+1}\right)\ \forall i \in \mathcal{H}_t \cap \mathcal{I}_t,\\
c_{i,t+1} &= (1-\beta)\, a_{i,i,t}\left(r^l_{t+1} + \lambda q^S_{i,t+1} - \frac{\theta}{(1-\theta)} g^T_{i,t+1}\right)\ \forall i \in \mathcal{L}_t \cap \mathcal{I}_t.
\end{aligned}$$

Using these guesses in (B.5) and (B.6), it is clear the latter conditions always hold.

[7] This applies in cases of separating equilibrium as well as in cases of pooling equilibrium where all firms with low-quality projects invest to the full capacity.

The stochastic discount factor $\beta c_{i,t}/c_{i,t+1}$ in the remaining Euler equations (B.1), (B.2), (B.3) and (B.4) can be rewritten as: $1/\Xi_{t+1} \equiv \beta c_{i,t}/c_{i,t+1}$, where

$$\begin{aligned}
\Xi_{t+1} \;=\; & \sum_{j\in\mathcal{H}_t\cap\mathcal{I}_t} \zeta_i^{hP} \frac{\hat{r}_{j,t+1}+\lambda q^S_{j,t+1}}{q^P_{j,t}} + \sum_{j\in\mathcal{L}_t\cap\mathcal{I}_t} \zeta_j^{lP} \frac{\hat{r}_{j,t+1}+\lambda q^S_{j,t+1}}{q^P_{j,t}} \\
& + \sum_{j\in\mathcal{H}_{t-1}\cap\mathcal{I}_{t-1}} \zeta_j^{hS} \frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^S_{j,t}} + \sum_{j\in\mathcal{L}_{t-1}\cap\mathcal{I}_{t-1}} \zeta_j^{lS} \frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^S_{j,t}} \\
& + \zeta^{hO} \frac{r^h_{t+1}+\lambda q^h_{t+1}}{q^h_t} + \zeta^{lO} \frac{r^l_{t+1}+\lambda q^l_{t+1}}{q^l_t}.
\end{aligned}$$

C Full-model solution in the deterministic steady state

In this appendix, I re-derive analytically selected propositions from Section 3 for the deterministic steady state of the full model.

C.1 Cases without binding SGC: first-best

If the SGC is not binding, only firms with high-quality investment opportunities invest and because of competition, the asset price is $q^h = 1$. Firms with low-quality projects do not invest because the separating condition

$$V_{i,t} \mid_{investing} < V_{i,t} \mid_{buying\; high-quality\; assets} \quad \forall i \in \mathcal{L}_t, \tag{C.1}$$

which collapses to $A^h > A^l$ is always satisfied. Due to logarithmic utility, firms always consume a $1-\beta$ fraction of their wealth: $c = (1-\beta)\, h \left(r^h + \lambda\right)$, which aggregates to $C = (1-\beta)\, H \left(r^h + \lambda\right)$.

Combining the good market clearing condition $X = Y - C = Hr^h - C$ with the law of motion for capital $X = (1-\lambda)\, H$, I obtain:

$$\begin{aligned}
Hr^h - C &= (1-\lambda)\, H \\
Hr^h - (1-\beta)\, H \left(r^h + \lambda\right) &= (1-\lambda)\, H, \\
r^h + \lambda &= \frac{1}{\beta}.
\end{aligned}$$

C.2 Cases with binding SGC

The SGC is binding when it restricts investment of firms with high-quality projects, i.e. $a^P_{i,i,t} = (1-\theta)\, x_{i,t}\ \forall i \in \mathcal{H}_t$. Their budget constraints (4.2) become

$$c_{i,t} + \left(1 - \theta q^h_t\right) x_{i,t} = w_{i,t}\ \forall i \in \mathcal{H}_t. \tag{C.2}$$

Substituting for $c_{i,t}$ from (B.7) in (C.2), I get their investment function $x^h_{i,t} = \beta w_{i,t} / \left(1 - \theta q^h_t\right)\ \forall i \in \mathcal{H}_t$, which aggregates to $X^H_t = \pi\mu\beta W_t / \left(1 - \theta q^h_t\right)$. Combining it with the aggregate version of (4.3), $W_t = H_t(r^h + \lambda q^h)$, we can rewrite it in the steady state as

$$(1-\lambda)\left(1 - \theta q^h\right) = \pi\mu\beta\left(r^h + \lambda q^h\right). \tag{C.3}$$

The goods market clearing condition $Y_t = X_t + C_t$ becomes in the steady state:

$$r^h = (1-\lambda) + (1-\beta)\left(r^h + \lambda q^h\right). \tag{C.4}$$

Combining (C.3) with (C.4), I obtain the market price:

$$q^h = \frac{(1-\lambda)(1-\pi\mu)}{(1-\lambda)\theta + \pi\mu\lambda}. \tag{C.5}$$

The SGC is binding only if $q^h > 1$, i.e., when $(1-\lambda)(1-\pi\mu) > (1-\lambda)\theta + \pi\mu\lambda$ (which is equivalent to $(1-\theta) > \pi\mu/(1-\lambda)$). This leads to the next proposition.

Proposition 5. *If "skin in the game" is sufficiently large to be binding, i.e., θ is sufficiently low to satisfy*

$$1 - \theta > \frac{\pi\mu}{1-\lambda}, \tag{C.6}$$

then in the deterministic steady state the price of high-quality assets q^h exceeds 1.

I denote $R_{i,t+1}$ as the gross return on wealth: $R_{i,t+1} = w_{i,t+1}/w_{i,t}$. First, assume that the implicit recourse cannot be provided; and the explicit recourse is too costly ($\tau > \bar{\tau}$). Then the separating condition (C.1) can be rewritten as follows when condition (C.6) holds:[8]

[8]Note that if no recourse is provided, all assets reveal their quality in the period following their issuance, at the latest.

$$\begin{aligned} R_{i,t+1}\,|_{mimicking} &< R_{i,t+1}\,|_{buying\,high-quality\,assets} \;\; \forall i \in \mathcal{L}_t, \\ \frac{r^l + \lambda q^l}{\frac{1-\theta q^h}{1-\theta}} &< \frac{r^h + \lambda q^h}{q^h}, \\ q^l &< \frac{1-\theta q^h}{1-\theta}. \end{aligned}$$

Substituting for q^h from (C.5) and using $\frac{A^h}{q^h} = \frac{A^l}{q^l}$, I get:

$$\frac{A^h}{A^l} > \frac{(1-\theta)\,q^h}{1-\theta q^h} = \frac{(1-\theta)(1-\pi\mu)(1-\lambda)}{\pi\mu\,(\theta+\lambda(1-\theta))}. \tag{C.7}$$

The explicit recourse is provided in equilibrium when $q^h < (1+\tau)/(1+\theta\tau)$ (obtained similarly as in subsection A.1), which after substituting for q^h from (C.5) can be rewritten as $\tau < \bar{\tau}^F = \frac{1}{\lambda+(1-\lambda)\theta}\left(\frac{(1-\theta)(1-\lambda)}{\pi\mu} - 1\right)$. Then the separating condition (C.1) becomes:

$$\frac{r^h + \lambda q^h}{q^h} > \frac{(1-\theta)(r^l + \lambda q^l) - \theta\left(r^{EG} - r^l\right)(1+\tau)}{1-\theta q^G},$$

which can be simplified to

$$\frac{A^h}{A^l} > \frac{(1+\tau)(1-\lambda\rho_A)}{(1+\tau)(1-\lambda\rho_A) - \pi\mu\tau\,(1+\lambda\rho_B)}, \tag{C.8}$$

where $\rho_A = \frac{\theta(1+\tau)}{(1+\theta\tau)(\lambda+(1-\lambda)(1+\theta\tau)/(1+\tau))}$ and $\rho_B = \frac{1+\tau}{(1-\lambda)(1-\theta\tau)}$. Since (C.6) holds, the RHS of (C.7) exceeds one; and since $\tau > 1$, the RHS of (C.8) exceeds one too. Therefore, a pooling equilibrium exists for the low productivity dispersions without as well as with the provision of explicit recourse.

When I allow the implicit recourse to occur and apply the intuitive criterion to refine PBEs, the separating condition (C.1) can be summarized in the the following proposition.

Proposition 6. *Suppose condition* (C.6) *holds and the explicit recourse is too costly* $\tau > \tilde{\tau}$*. Then a separating equilibrium is possible in the deterministic steady state if and only if*

$$\frac{A^h}{A^l} > \frac{(1-\theta P)\,q^h}{1-\theta P q^h} = \frac{(1-\theta P)(1-\pi\mu)(1-\lambda)}{P\pi\mu\,(\theta+\lambda(1-\theta))}, \tag{C.9}$$

where $P \equiv \frac{q^P}{q^h} = \frac{\hat{r}+\lambda q^h}{r^h+\lambda q^h} > 1$ *is the price premium for the equilibrium implicit guarantee. This implies that the separating equilibrium is more likely in the presence of an implicit recourse,* *but since the RHS of* (C.9) *exceeds one, a pooling equilibrium exists for the low productivity dispersions which do not satisfy* (C.9).

Proof. Since $P > 1$ (see later in the proof), when comparing separating conditions (C.7) and (C.9), it is straightforward that condition (C.9) is satisfied for a larger parameter subspace.

I proceed by deriving the equilibrium solution and the condition (C.9) from the separating condition (C.1) and showing that the RHS of the inequality (C.9) exceeds one and is independent on A^h and A^l.

The steady-state conditions for the separating PBE, which satisfies the intuitive criterion, are as follows:

$$\text{Investment function:} \qquad (1-\lambda)\left(1-\theta q^P\right) = \pi\mu\beta\left(r^h + \lambda q^h\right), \tag{C.10}$$

$$\text{Goods market clearing:} \qquad r^h = (1-\lambda) + (1-\beta)\left(r^h + \lambda q^h\right), \tag{C.11}$$

$$\text{Asset market clearing:} \qquad \frac{\hat{r} + \lambda q^h}{q^P} = \frac{r^h + \lambda q^h}{q^h}, \tag{C.12}$$

$$\text{Binding non-default cond.:} \qquad V^{ND}\left(w' \mid_{\chi'=1}\right) = V^D\left(w' \mid_{\chi'=0}\right). \tag{C.13}$$

Using the following property given by the logarithmic utility function:

$$V(w) = \log\left((1-\beta)\, w\right) + \beta \log\left((1-\beta)\,\beta R w\right) + \beta^2 \log\left((1-\beta)\,\beta^2 R^2 w\right) + \ldots = \frac{1}{1-\beta}\log(w) + V(1),$$

I write firms' value functions in the following way:

$$\begin{aligned}
V^D\left(w' \mid_{\chi'=0}\right) &= V^D(1) + \frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h + \lambda q^h\right)}{(1-\theta q^P)} w\right) \\
V^{ND}\left(w' \mid_{\chi'=1}\right) &= V^{ND}(1) + \frac{1}{1-\beta}\log\left(\beta\frac{(1-\theta)\left(r^h + \lambda q^h - \frac{\theta}{1-\theta}\left(r^{IG} - r^h\right)\right)}{(1-\theta q^P)} w\right).
\end{aligned}$$

Value functions with unitary wealth can be obtained as follows:

$$\begin{aligned}
V^{ND}(1) &= \log(1-\beta) + \beta\left(\pi\mu V^{ND}\left(\beta R^{h,ND}\right) + \pi(1-\mu)\, V^{ND}\left(\beta R^l\right) + (1-\pi)\, V^{ND}\left(\beta R^z\right)\right) \\
&= \log(1-\beta) + \beta\left(\frac{\pi\mu\log\left(\beta R^{h,ND}\right)}{1-\beta} + \pi(1-\mu)\frac{\log\left(\beta R^l\right)}{1-\beta} + (1-\pi)\frac{\log\left(\beta R^z\right)}{1-\beta} + V^{ND}(1)\right) \\
&= \frac{\log(1-\beta)}{1-\beta} + \frac{\beta\log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,ND}\right) + \pi(1-\mu)\log\left(R^l\right) + (1-\pi)\log\left(R^z\right)\right). \\
V^D(1) &= \frac{\log(1-\beta)}{1-\beta} + \frac{\beta\log(\beta)}{(1-\beta)^2} + \frac{\beta}{(1-\beta)^2}\left(\pi\mu\log\left(R^{h,D}\right) + \pi(1-\mu)\log\left(R^l\right) + (1-\pi)\log\left(R^z\right)\right),
\end{aligned}$$

where $R^{h,ND}, R^{h,D}$ are one-period returns on wealth for firms with high-quality projects that have never defaulted on implicit recourse and for those that have defaulted on implicit recourse, respectively. R^l and R^z are one period returns on wealth for firms with low-quality projects and no projects, respectively. Substituting the above equations into the non-default condition (C.13) and canceling the terms equal for both value functions, I obtain:

$$\log\left(\beta\left(1-\theta\right)\left(r^{h}+\lambda q^{h}-\frac{\theta}{1-\theta}\left(r^{IG}-r^{h}\right)\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,ND}\right)$$
$$=\log\left(\beta\left(1-\theta\right)\left(r^{h}+\lambda q^{h}\right)\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right),$$

where the LHS shows the utility from consumption when wealth is reduced by repayment of the implicit recourse and from the future discounted benefit of having a good reputation. The RHS, then, shows higher immediate utility from savings on the implicit recourse, but the future utility is lower, since the firm can no longer issue and sell new loans. This equation can further be simplified using (C.12) and substituting for the returns:

$$\begin{aligned}
-\log\left(\frac{r^{h}+\lambda q^{h}-\theta\left(r^{IG}+\lambda q^{h}\right)}{\left(1-\theta\right)\left(r^{h}+\lambda q^{h}\right)}\right) &= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,ND}}{R^{h,D}}\right)\\
&= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{\left(1-\theta\right)\left(r^{h}+\lambda q^{h}-\frac{\theta}{1-\theta}\left(r^{IG}-r^{h}\right)\right)}{\left(1-\theta q^{P}\right)}\frac{1}{\left(r^{h}+\lambda q^{h}\right)}\right)\\
&= \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{r^{h}+\lambda q^{h}-\theta\left(r^{IG}+\lambda q^{h}\right)}{r^{h}+\lambda q^{h}-\theta q^{h}\left(r^{IG}+\lambda q^{h}\right)}\right).
\end{aligned}$$

Finally, this non-default condition can be expressed as follows:

$$\log\left(\frac{1-\theta}{1-\theta P}\right) = \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta P}{1-\theta P q^{h}}\right). \tag{C.14}$$

The LHS is the ratio of immediate utility from defaulting to immediate utility from honoring the implicit recourse. The RHS is the ratio of the discounted sum of future utilities from honoring the recourse to the discounted sum of future utilities from defaulting on recourse. Note that the argument of the logarithm on the RHS corresponds to $R^{h,ND}/R^{h,D}$ and exceeds one only if $q^h > 1$. Indeed only then is securitization profitable. Equation (C.14) also implies that only if $q^h > 1$, is the RHS positive and as a result only then $P > 1$. This can be intuitively interpreted that only when securitization is profitable, firms suffer from punishment and they can provide a credible implicit recourse.

The steady-state condition (C.14) together with the following condition (obtained by combining C.10 and C.11)

$$q^{h}=\frac{\left(1-\lambda\right)\left(1-\pi\mu\right)}{\left(1-\lambda\right)\theta P+\pi\mu\lambda} \tag{C.15}$$

determine the solution to q^h and P, which depends only on the time preference (β), depreciation (λ) and financial frictions parameters (π, μ, θ). Therefore, q^h and P do not depend on the productivity levels A^h and A^l, which is the first step of the proof. Note also that if condition (C.6) holds, condition (C.15) implies that $q^h > 1$.

The second step is to derive (C.9) from (C.1). Using similar transformations as with condition (C.13), I rewrite the separation condition (C.1):[9]

$$\begin{aligned}
\log\left(\frac{\beta\left(1-\theta\right)\left(r^{l}+\lambda q^{l}\right)}{\left(1-\theta q^{P}\right)}\right)+\frac{\beta\pi\mu}{1-\beta}\log\left(R^{h,D}\right) &< \log\left(\beta\frac{\left(r^{h}+\lambda q^{h}\right)}{q^{h}}\right)+\beta\pi\mu\log\left(R^{h,ND}\right)\\
\log\left(\frac{\left(r^{l}+\lambda q^{l}\right)\left(1-\theta\right)}{\left(1-\theta q^{P}\right)}\frac{q^{h}}{\left(r^{h}+\lambda q^{h}\right)}\right) &< \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{R^{h,ND}}{R^{h,D}}\right)\\
\log\left(\frac{\left(1-\theta\right)q^{l}}{\left(1-\theta P q^{h}\right)}\right) &< \frac{\beta\pi\mu}{1-\beta}\log\left(\frac{1-\theta P}{1-\theta P q^{h}}\right).
\end{aligned}$$

I substitute the RHS of the above condition using (C.14) to get:

$$\begin{aligned}
\log\left(\frac{\left(1-\theta\right)q^{l}}{\left(1-\theta P q^{h}\right)}\right) &< \log\left(\frac{1-\theta}{1-\theta P}\right)\\
\frac{A^{h}}{A^{l}} &> \frac{\left(1-\theta P\right)q^{h}}{1-\theta P q^{h}}=\frac{\left(1-\theta P\right)\left(1-\pi\mu\right)\left(1-\lambda\right)}{P\pi\mu\left(\theta+\lambda\left(1-\theta\right)\right)}. \qquad \text{(C.16)}
\end{aligned}$$

The equality in (C.16) is obtained by substituting for q^h from (C.15). The RHS of (C.16) exceeds one, because when condition (C.6) holds, $q^h > 1$ and $P > 1$ (see earlier in the proof).

Finally, for simplicity we have focused on cases, where the explicit recourse will not be provided, which takes place when

$$\frac{\partial V'}{\partial r^{EG\prime}}=\frac{V'}{w'}\frac{\partial}{\partial r^{EG\prime}}\frac{\left(r^{h}+\lambda q^{h}\right)\left(1-\theta\right)-\theta\max\left\{r^{EG\prime}+\Delta^{IG\prime}-r^{h},0\right\}-\theta\max\left\{r_{i}^{EG\prime}-r^{h},0\right\}\tau}{1-\theta q_{i}}w_{i}<0,$$

which can be rewritten as $\tau < \tilde{\tau}^{F} = \frac{1+d\Delta^{IG\prime}/dr_{i}^{EG\prime}}{P(\lambda+(1-\lambda)\theta)}\left(\frac{(1-\lambda)(1-\theta P)}{\pi\mu}-(1+\lambda(P-1))\right)$.

D Numerical solution of the fully stochastic dynamic model

To capture the effect of switching between a separating and a pooling equilibrium, I use global numerical approximation methods for the model solution. In particular, I find the numerical approximation for endogenous variables on a grid of state variables $\bar{S}=(A,K,\omega)$ by iteration.

The model solution can be reduced to finding the solution for the set of three non-predetermined endogenous variables $q^{h}\left(\bar{S}\right)$, $q^{l}\left(\bar{S}\right)$ and $V^{diff}\left(\bar{S}\right)\equiv V^{ND}\left(\bar{S}\right)-V^{D}\left(\bar{S}\right)$, which I denote $\bar{\Gamma}\left(\bar{S}\right)=\left\{q^{h},q^{l},V^{diff}\right\}\mid\bar{S}$. Expectations about their next-period values determine the current level of all endogenous variables. Once I know $\bar{\Gamma}\left(\bar{S}\right)$, I can find the remaining endogenous variables, including the law of motion for the endogenous state variables K and ω. Therefore, all equilibrium conditions can be written as $E\left(\bar{\Gamma},\bar{\Gamma}',\bar{S}'\left(\bar{\Gamma}\right)\right)\mid\bar{S}=0$.

I use the following algorithm to find the numerical approximation to the model solution.

[9] Recall that in the separating equilibrium, selected by the intuitive criterion, mimicking firms with low-quality projects choose to default on implicit recourse, since $r^{IG} > r^{IG}_{l,cred,s}$.

Initiation: I construct a three-dimensional grid $\mathbb{S}$ of state variables as all possible combinations of $\bar{A}, \bar{K}$ and $\bar{\omega}$, which are vectors of selected nodes for state variables. Since aggregate productivity takes only two values, I choose $\bar{A} = \left\{A^H, A^L\right\}$. Vector $\bar{K}$ consists of n equidistant values for K with the median being the steady-state value of K. Vector $\bar{\omega}$ consists of n equidistant values from the interval of possible values for ω in equilibrium, i.e., from $[\mu, 1]$.

I make an initial guess for the value function difference on the grid: $\bar{V}_0^{diff}(\mathbb{S}) = \left\{V_0^{diff}\right\} \mid \bar{S} \in \mathbb{S}$. I choose the stopping criterion $\varepsilon > 0$ and set the value function iteration counter to zero, $l = 0$.

Step 1: I do nested iteration to find out the values of $\left\{q^h, q^l\right\} \mid \bar{S} \in \mathbb{S}$ for the particular guess of the value function difference V_l^{diff}.[10] I make an initial guess for the remaining non-predetermined endogenous variables of interest on the grid $\left\{\bar{q}_0^h(\mathbb{S}), \bar{q}_0^l(\mathbb{S})\right\} = \left\{q_0^h, q_0^l\right\} \mid \bar{S} \in \mathbb{S}$ and set the price iteration counter to zero, $k = 0$.

- **Step 1a:** For all combinations of state variables on the grid $\forall \bar{S} \in \mathbb{S}$, I compute V_{l+1}^{diff} and q_{k+1}^h, q_{k+1}^l, which satisfy[11]

  $$E\left(\bar{\Gamma}_{l+1,k+1}, \bar{\Gamma}'_{l,k}, \bar{S}'\left(\bar{\Gamma}_{l,k}\right)\right) \mid \bar{S} = 0.$$

  Note that $\bar{\Gamma}'_{l,k}$ is a function of $\bar{S}'\left(\bar{\Gamma}_{l,k}\right)$, which might be in between the grid points. In this case, I use linear interpolation on the values of the neighboring grid points on the state space.

- **Step 1b:** If the difference between the values of the two subsequent iterations for prices is smaller than the stopping criterion, i.e., if

  $$\| \bar{q}_{k+1}^h(\mathbb{S}) - \bar{q}_k^h(\mathbb{S}) \| + \| \bar{q}_{k+1}^l(\mathbb{S}) - \bar{q}_k^l(\mathbb{S}) \| < \varepsilon,$$

  then I move to Step 2; otherwise, I go back to Step 1a with the price iteration counter k increased by one.

Step 2: If the difference between the values of the two subsequent iterations for the difference in value functions is smaller than the stopping criterion, i.e., if $\| \bar{V}_{l+1}^{diff}(\mathbb{S}) - \bar{V}_l^{diff}(\mathbb{S}) \| < \varepsilon$, then I move to Step 3; otherwise, I go back to Step 1 with the iteration counter l increased by one.

[10] The subscript for V_l^{diff} denotes the value function iteration number.

[11] The first subscript of $\bar{\Gamma}$ denotes the number of the value function iteration and the second subscript, the number of the price function iteration: $\bar{\Gamma}_{l+1,k+1} = \left\{q_{k+1}^h, q_{k+1}^l, V_{l+1}^{diff}\right\}$.

Step 3: I declare $\bar{\Gamma}_{l,k}(\mathbb{S})$ the final approximate solution and compute the remaining endogenous variables in the model.

E Appendix to the empirical analysis

Data summary statistics. Table E.1 shows the summary statistics for the variables used from the PDS database before and after Winsorizing and controlling for trend and persistence in the first-stage regression.

First-stage regression. To avoid potential spurious regression problem I clear *Overcollat* from potential trend and persistence in a first-stage regression, which I run on the level of individual deals (for every i) to account for heterogeneity:

$$Overcollat_{i,t} = \alpha_i^{FS} + \beta_i^{FS}\, Overcollat_{i,t-1} + \gamma_i^{FS}\, Deal\, age_{i,t} + \epsilon_{i,t}\ \forall i,$$

and I use residuals from the regression equation $Overcollat_{i,t}^{D} \equiv \epsilon_{i,t}\ \forall i$ in the second-stage (eq. 2.1).

Robustness checks. First, I run the same regression on the sub-sample excluding the late-2000s crisis and post-crisis period. Results in Table E.2 are comparable to those in Table 1, which suggests that the crisis episode does not determine the results.

Second, I Winsorize both delinquency rates and the overcollateralization rates at the 2.5%-level to account for potential data errors and limit the effect of potential outliers. The regression results reported in Table E.4 are qualitatively similar to the results in Table 1.

Third, I show that the results are not driven by controlling for trend and persistence of the $Overcollat^{D}$ variable. I use the actual *Overcollat* directly in the main regression of interest (eq. 2.1) and obtain results reported in Table E.5, which are qualitatively similar to those in Table 1.

Finally, I show in Table E.3 that when the regression is run on the subset of deals issued in the boom stage of the business cycle, I cannot find support for the signaling relationship in the UK. This result is in line with the hypothesis of weaker or non-existent signaling for deals issued in the boom stage.

Table E.1. Data summary statistics

	Definition	Transf.	Country	(1) Obs.	(2) Mean	(3) Std. Dev.	(4) Min	(5) Max
DelinqRate	Percentage ratio of receivables 90 or more days past due to the original pool balance	-	UK, SP, NL	14,508	1.75	3.25	0	26.65
			UK	4,373	4.55	4.54	0	26.65
			SP	5,933	0.53	0.93	0	13.41
		Winsorized	UK, SP, NL	14,508	1.67	2.89	0	11.96
			UK	4,373	4.28	3.88	0	11.96
			SP	5,933	0.53	0.92	0	11.96
Overcollat	Percentage difference between the principal value of collateral asset and the principal value of deal tranches normalized by the original collateral principal value	-	UK, SP, NL	14,991	-0.47	4.63	-117.48	90.64
			UK	4,564	0.18	5.08	-99.71	52.47
			SP	6,009	-0.63	2.16	-32.04	34.21
		Winsorized	UK, SP, NL	14,991	-0.42	1.34	-6.18	1.87
			UK	4,564	-0.18	1.38	-6.18	1.87
			SP	6,009	-0.57	1.06	-6.18	1.87
		detrended	UK, SP, NL	14,349	-0.00	1.23	-31.12	72.77
			UK	4,360	0.00	0.75	-21.66	21.64
			SP	5,781	0.00	1.07	-26.63	24.45
		Winsorized & detrended	UK, SP, NL	14,349	0.00	0.32	-1.15	1.00
			UK	4,360	-0.00	0.26	-1.15	1.00
			SP	5,781	0.01	0.32	-1.15	1.00

Table E.2. Regression on pre-crisis subsample[a]

Countries	(1) UK, SP, NL	(2) UK	(3) SP	(4) NL
$DelinqRate_{i,t-1}$	0.841***	0.766***	0.411*	0.681***
	(0.022)	(0.027)	(0.213)	(0.033)
$Overcoll^{D}_{i,t-1}$	0.000	-0.109**	-0.001	-0.003
	(0.003)	(0.054)	(0.001)	(0.002)
$Overcoll^{D}_{i,t-1} \times$	0.004	0.122**	-0.001	0.002
$D^{origin\,in\,boom}_{i}$	(0.004)	(0.056)	(0.003)	(0.003)
$Deal\,age_{i,t}$	-0.011***	-0.050***	-0.003***	-0.002***
	(0.003)	(0.011)	(0.001)	(0.001)
$Overcoll^{D}_{i,t-1} \times$	-0.006*	0.027	0.002	0.000
$D^{boom}_{i,t}$	(0.003)	(0.061)	(0.002)	(0.003)
$Output\,gap$	5.306			
	(3.925)			
Observations	3,860	1,010	1,584	1,266
R-squared	0.691	0.762	0.254	0.579
Number of deals	338	115	118	105

[a] Time period excludes the late-2000s financial crisis and the following period. For the whole sample of countries the time period is 1998Q2-2007Q2; 2000Q2-2007Q2 for the UK; 1998Q3-2007Q2 for Spain; and 1998Q2-2013Q2 for the Netherlands.

Table E.3. Regression on loans issued in boom

Countries	(1) UK, SP, NL	(2) UK	(3) SP	(4) NL
$DelinqRate_{i,t-1}$	0.864***	0.861***	0.826***	0.896***
	(0.010)	(0.010)	(0.018)	(0.022)
$Overcoll^{D}$	0.026**	0.006	0.029	0.040
	(0.011)	(0.010)	(0.018)	(0.032)
$Deal\,age$	-0.009***	-0.023***	-0.001	-0.000
	(0.001)	(0.004)	(0.001)	(0.001)
$Overcoll^{D} \times$	-0.019*	0.039	-0.008	-0.041
D^{boom}	(0.011)	(0.049)	(0.021)	(0.033)
$Output\,gap$	-15.530***			
	(2.926)			
Observations	6,574	2,459	2,420	1,695
R-squared	0.859	0.895	0.758	0.855
Number of deals	325	112	111	102

Table E.4. Regression on Winsorized data

	(1)	(2)	(3)	(4)
Countries	UK, SP, NL	UK	SP	NL
$DelinqRate^{W}_{i,t-1}$	0.898***	0.872***	0.831***	0.885***
	(0.007)	(0.009)	(0.023)	(0.014)
$Overcoll^{D,W}_{i,t-1}$	-0.026*	-0.126*	-0.016	-0.004
	(0.015)	(0.070)	(0.023)	(0.007)
$Overcoll^{D,W}_{i,t-1}\times$	0.092***	0.220***	0.122**	-0.014
$D^{origin\,in\,boom}_{i}$	(0.031)	(0.083)	(0.056)	(0.009)
$Deal\,age_{i,t}$	-0.004***	-0.022***	-0.000	-0.000
	(0.001)	(0.004)	(0.000)	(0.000)
$Overcoll^{D,W}_{i,t-1}\times$	0.007	0.022	0.013	0.001
D^{boom}	(0.021)	(0.088)	(0.034)	(0.009)
$Output\,gap_{i,t}$	-5.063***			
	(1.478)			
Observations	13,226	3,949	5,486	3,791
R-squared	0.864	0.899	0.738	0.918
Number of deals	612	190	227	195

Table E.5. Without first-stage regression

	(1)	(2)	(3)	(4)
Countries	UK, SP, NL	UK	SP	NL
$DelinqRate_{i,t-1}$	0.858***	0.835***	0.807***	0.895***
	(0.009)	(0.012)	(0.022)	(0.012)
$Overcoll_{i,t-1}$	-0.005	-0.046***	-0.001	0.009
	(0.009)	(0.011)	(0.003)	(0.006)
$Overcoll_{i,t-1}\times$	0.030*	0.046**	0.042***	0.032
$D^{origin\,in\,boom}_{i}$	(0.017)	(0.021)	(0.015)	(0.030)
$Deal\,age_{i,t}$	-0.007***	-0.034***	-0.001	-0.002
	(0.001)	(0.004)	(0.001)	(0.003)
$Overcoll_{i,t-1}\times$	-0.011	0.004	0.001	-0.020
$D^{boom}_{i,t}$	(0.009)	(0.013)	(0.006)	(0.017)
$Output\,gap$	-6.610***			
	(2.173)			
Observations	13,820	4,141	5,703	3,976
R-squared	0.845	0.885	0.734	0.883
Number of deals	618	194	227	197